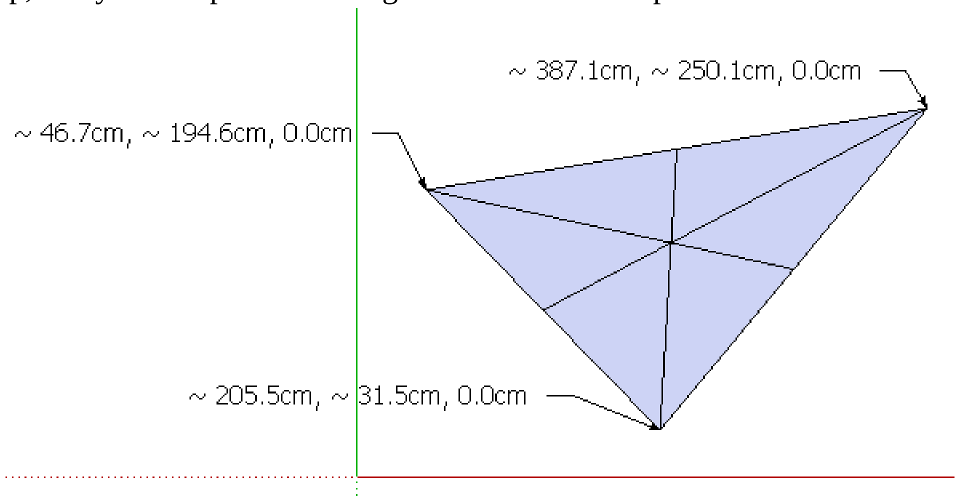
Medians and Centroid of a Triangle, in Google SketchUp

The centroid of a triangle is where the three medians intersect. This project will show you how to find the centroid in SketchUp, and you'll explore several geometric relationships related to the centroid and medians.



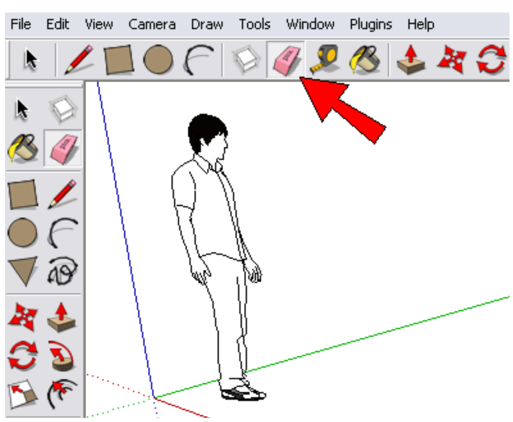
Teacher Note: All text that appears in **red** is for the teacher version only, and does not appear in the student version.

For this project, it helps to have some basic knowledge of Google SketchUp (though detailed instructions are provided). In particular, it's important to know how to zoom and pan the view. If you need more information on how to get started, and a description of some basic tools, please read 3DVinci's Getting Started Guide (PDF).

PC users: go to http://www.3dvinci.net/SketchUp_Intro_PC.pdf.
Mac users: go to http://www.3dvinci.net/SketchUp_Intro_MAC.pdf.

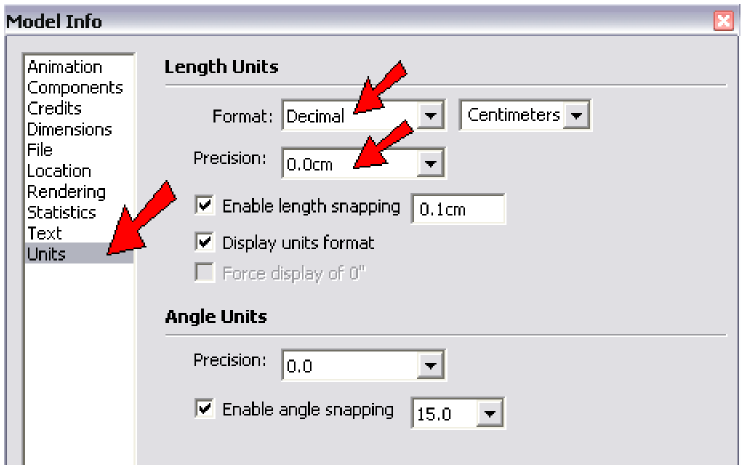
Triangle and Medians

1. Open Google SketchUp. If your file contains a person standing on the ground near the origin, click the **Eraser** tool and erase him.

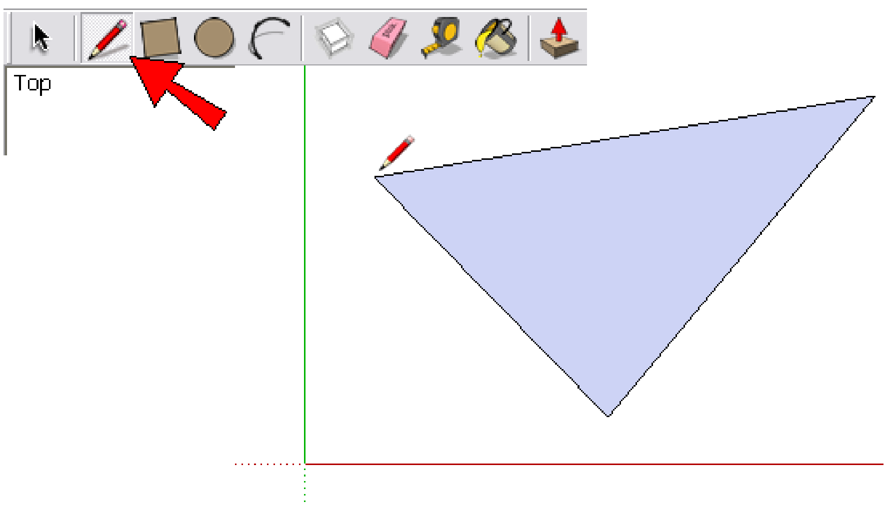


2. From the main menu, choose **Camera / Standard Views / Top**. Now you're looking down on the "ground," and the word **Top** appears in the top left corner of the SketchUp window.

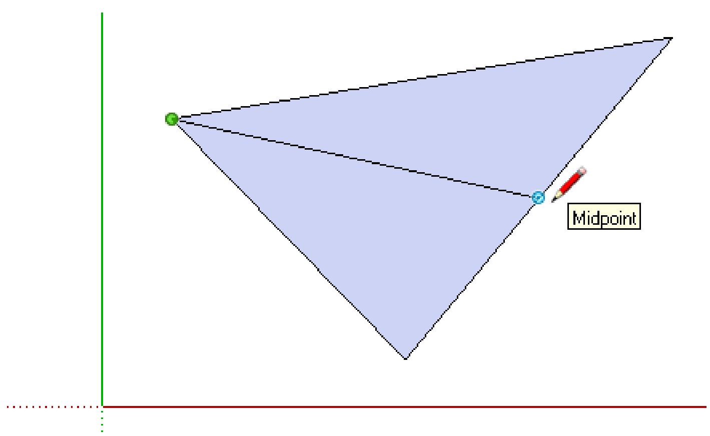
3. We'll be checking area and length measurements later, so we'll set the units to be easy to work with (in other words, no feet and inches). From the main menu choose **Window / Model Info**, and open the **Units** page. Set the format to **Decimal**, and use any measurement you like. Set the **Precision** so that you won't see too many decimal places. When you're done, close the **Model Info** window.



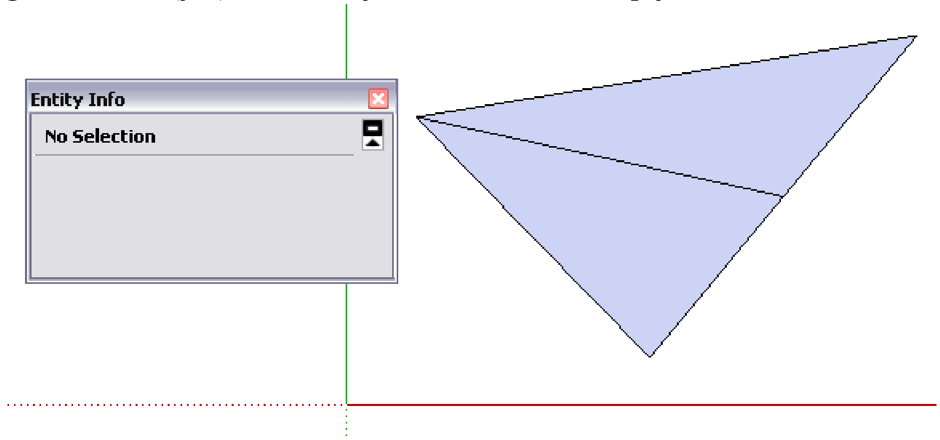
4. Click the **Line** tool and draw any three lines to create a triangle (be sure to end at the same point where you started). This project will work with any triangle, so yours can look completely different than this one.



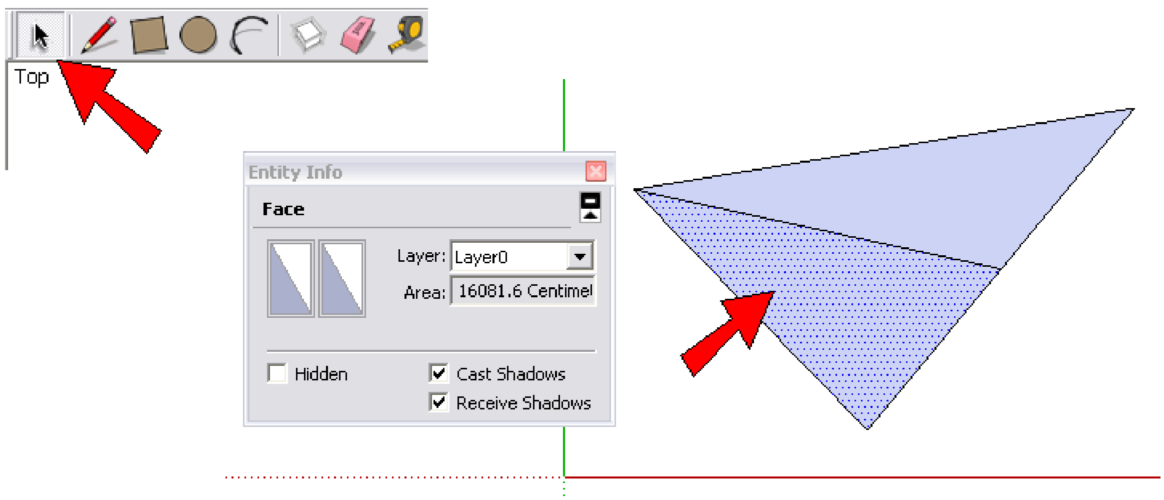
5. A triangle has three medians: lines that extend from each corner to the midpoint of the opposite edge. Draw a median line starting from any corner, ending at the opposite edge where you see the cyan dot and the "Midpoint" popup.



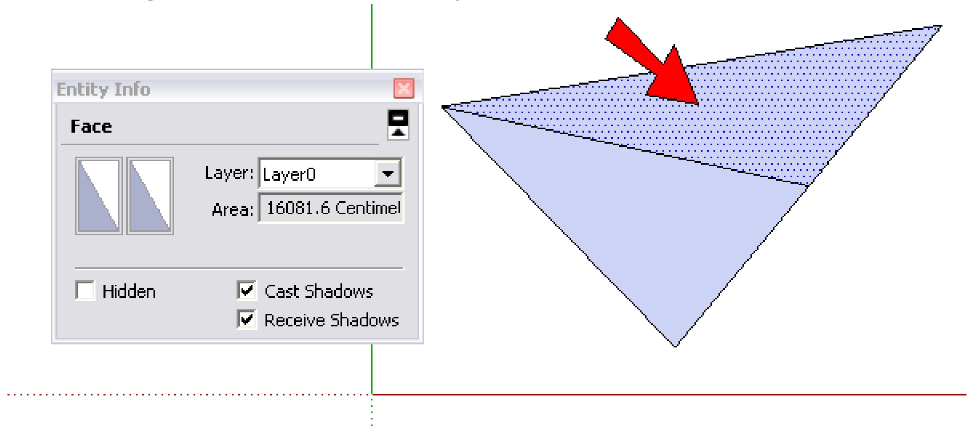
6. It may not look like it, but this median has divided the triangle into two triangles of equal area. To verify this, we'll take the area measurement of both triangles. From the main menu, choose **Window / Entity Info**. Because nothing is selected (yet), the **Entity Info** window is empty.



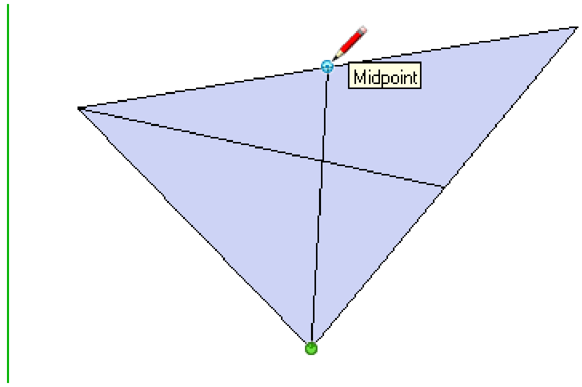
7. Activate the **Select** tool and click one of the two triangles to select it. The triangle's area is listed in the **Entity Info** window.



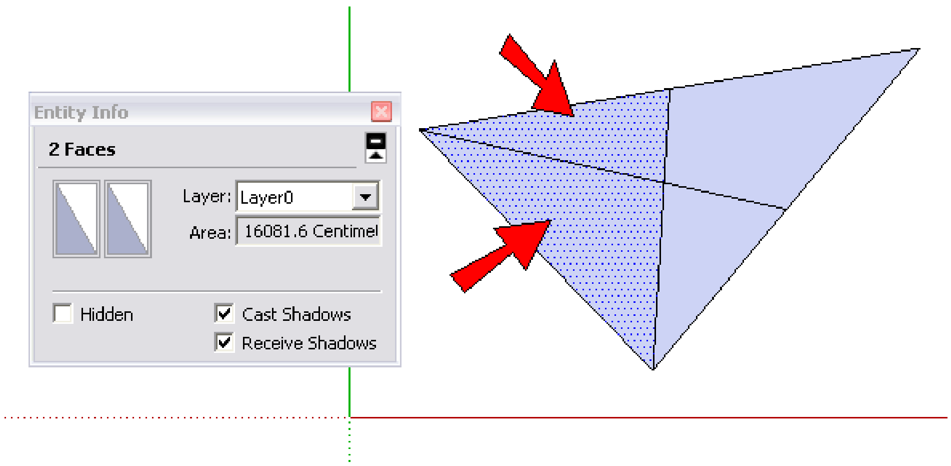
8. Now click the other triangle, whose area is exactly the same.



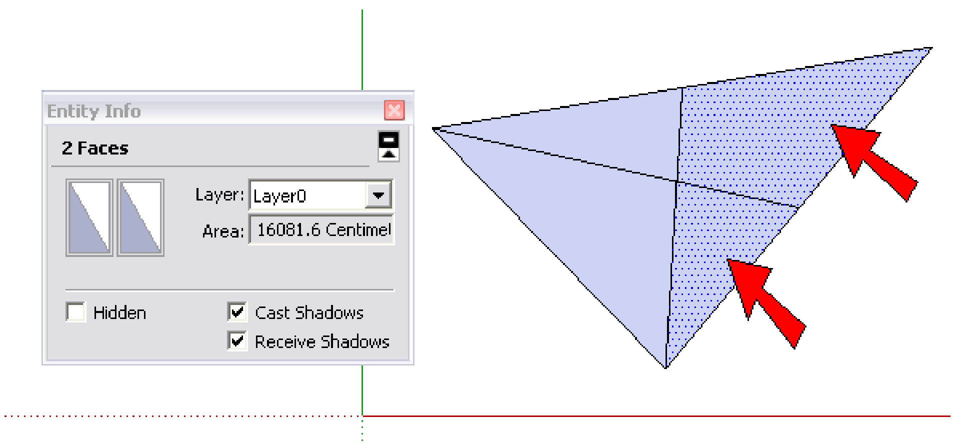
9. Use the **Line** tool again to draw another median.



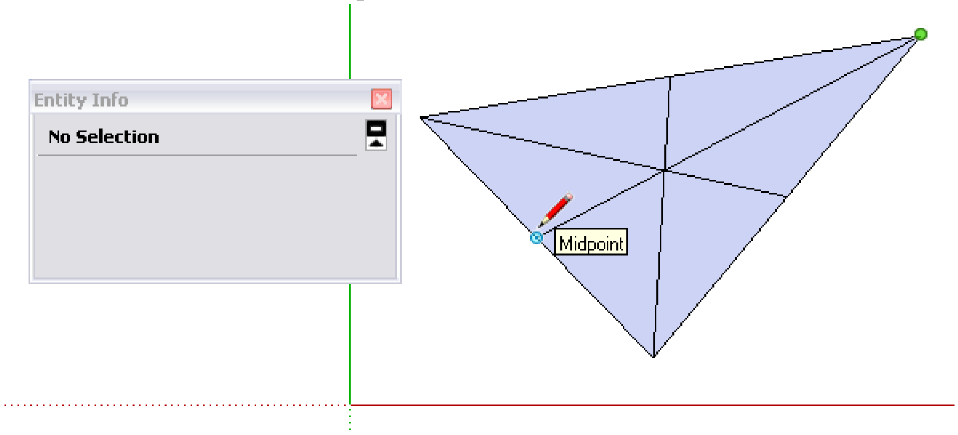
10. The two triangles on either side of this new median have the same area as the triangles on either side of the first median you drew. To check this, you'll need to select both triangles on either side of the new median line; pressing Shift with the **Select** tool enables you to select multiple objects.



11. Check the area on the other side as well.

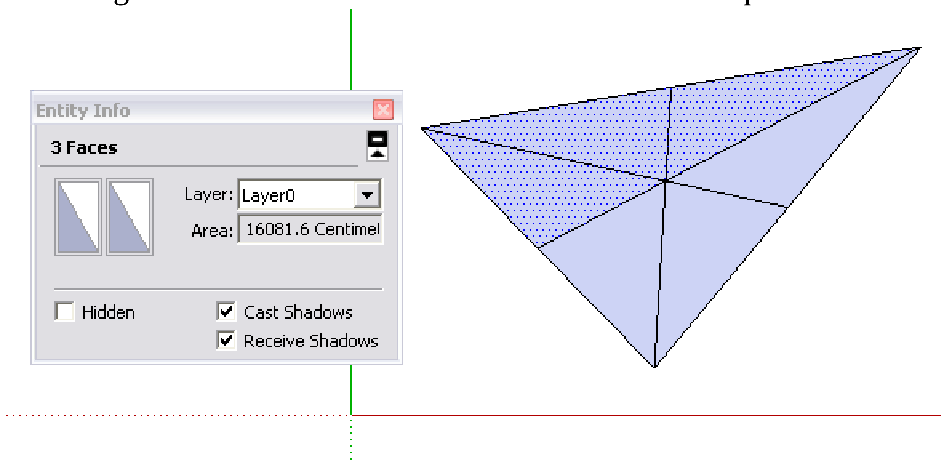


12. Now draw the third and last median line. It intersects the other two medians at the point where they themselves intersect, and this intersection point is called the centroid.



The centroid is also called the center of gravity, or the center of mass. Imagine the triangle made of paper, lying flat. If you held a pencil vertically, point up, and placed the triangle so that the centroid rested on the pencil point, the triangle would be perfectly balanced.

13. And of course, the triangle areas on either side of this third median are equal.



14. If you check each of the six small triangles individually, you'll see that they all have the same area.

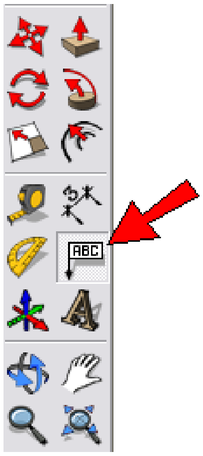
Where is the Centroid?

The coordinates of the centroid are the average of the coordinates of the three triangle corners. In other words, if the triangle corners have coordinates (x1,y1), (x2,y2), and (x3,y3), then the centroid coordinates are:

(x1+x2+x3)/3, (y1+y2+y3)/3

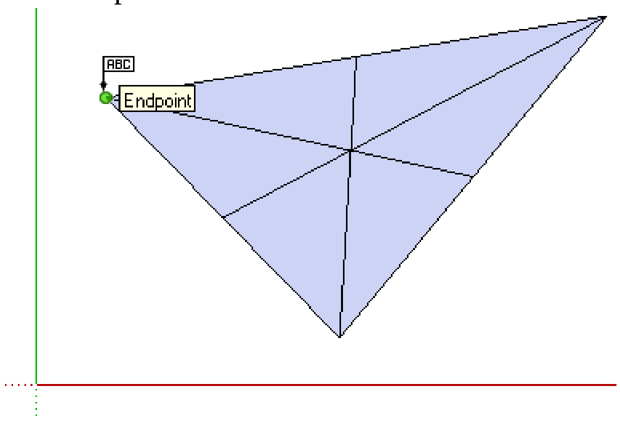
Let's use SketchUp to check this.

1. Activate the **Text** tool, which you can find on the **Large Tool Set** toolbar.

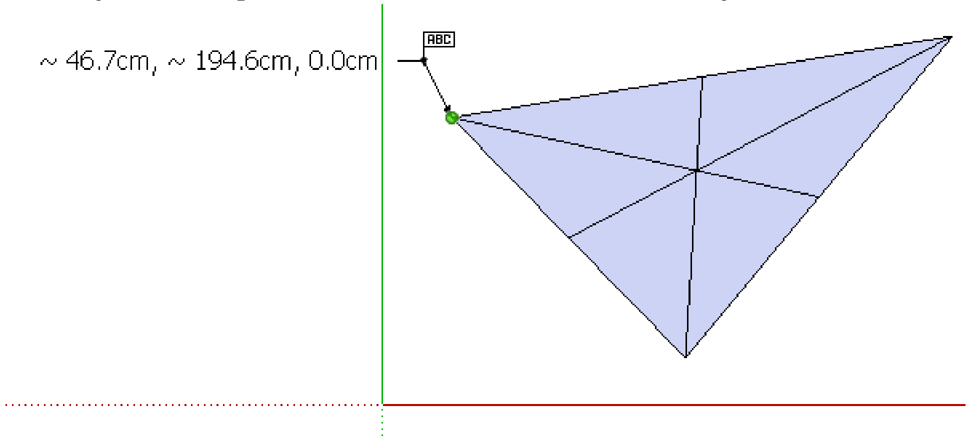


To display this toolbar, choose ***View / Toolbars / Large Tool Set*** *(PC) or* ***View / Tool Palettes / Large Tool Set*** *(Mac).*

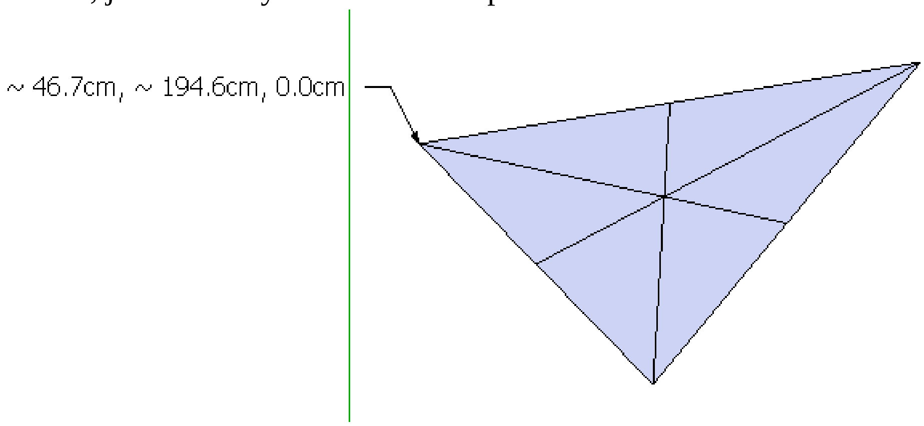
2. This tool is generally used to create labels, but when you click a point, the default label text is the point's coordinates. Click one of the corner points.



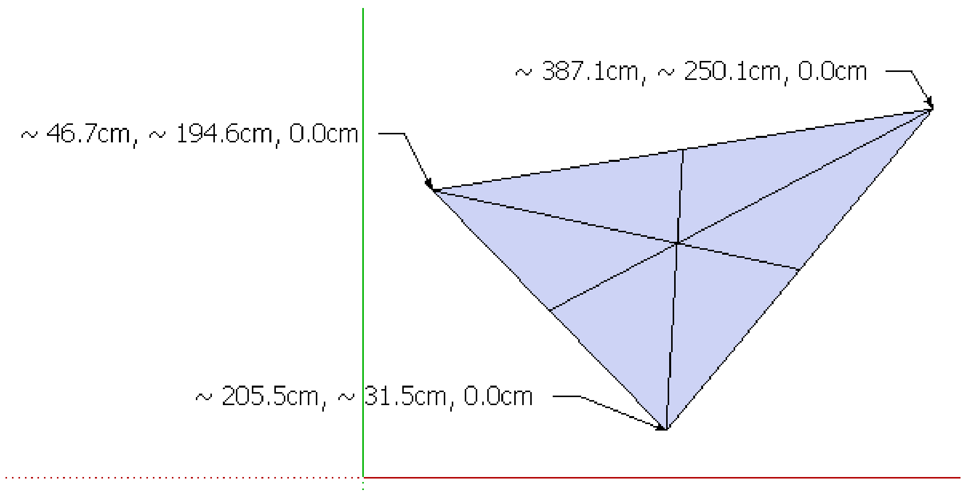
3. Move the mouse away from the point, until the coordinates are where you want them. Then click again.



4. If you were creating a label here, you would overwrite the coordinates with the label text. But since we want to keep the coordinates, just click anywhere in blank space.



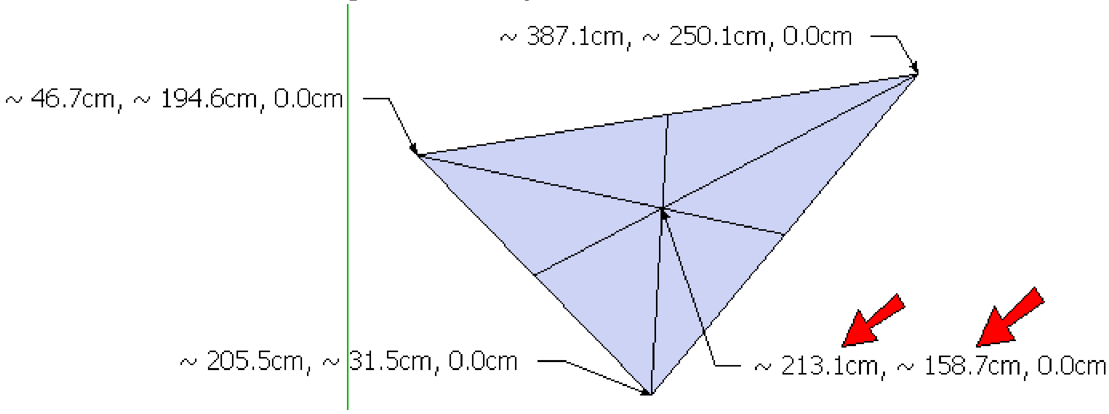
5. Add the coordinate labels for the other two corners.



For this triangle, the centroid coordinates should be the average values of the three X and Y coordinates of the corners:

(205.5 + 46.7 + 387.1)/3 , (31.5 + 194.4 + 250.1)/3, or (213.1, 158.7)

6. Add one more label to the centroid point, to verify that the coordinates are correct.

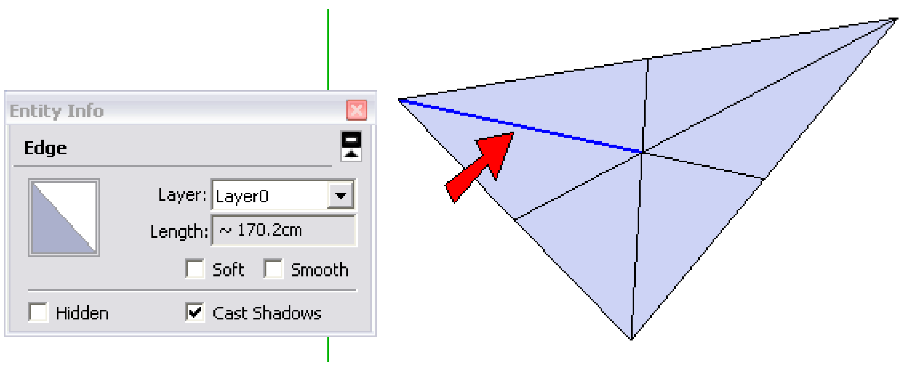


This example showed coordinates that all have positive X and Y values. Remember, if you have negative coordinates, ***subtract*** *them from the other coordinates when calculating the average!*

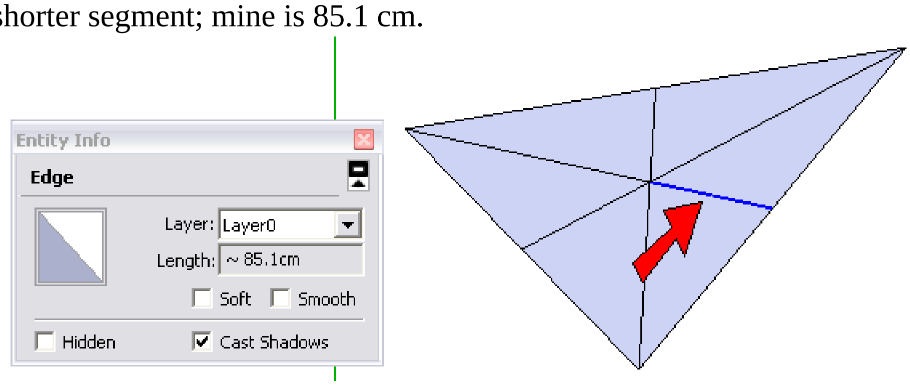
Segments of the Medians

Each of the three medians is divided at the centroid into two segments. And there is an interesting relationship between these segments.

1. We used the **Entity Info** window before to measure area, and it can also be used to measure length. Activate the **Select** tool and click one of the median segments, on the side where it meets the triangle corner. The length of the segment shown below is 170.2 cm.

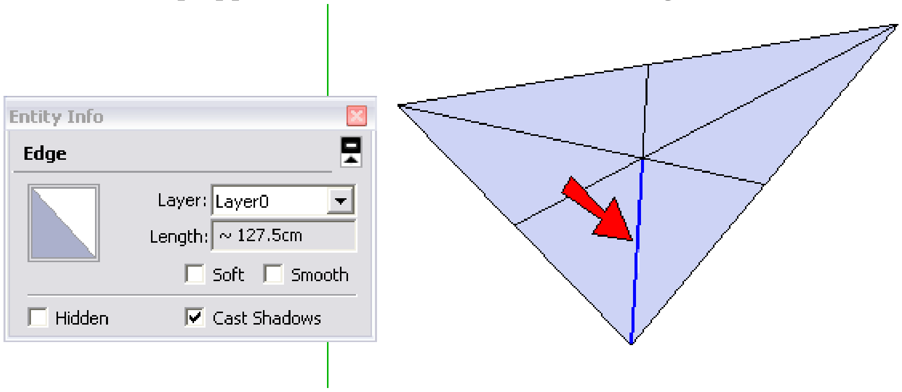


2. Now check the shorter segment; mine is 85.1 cm.

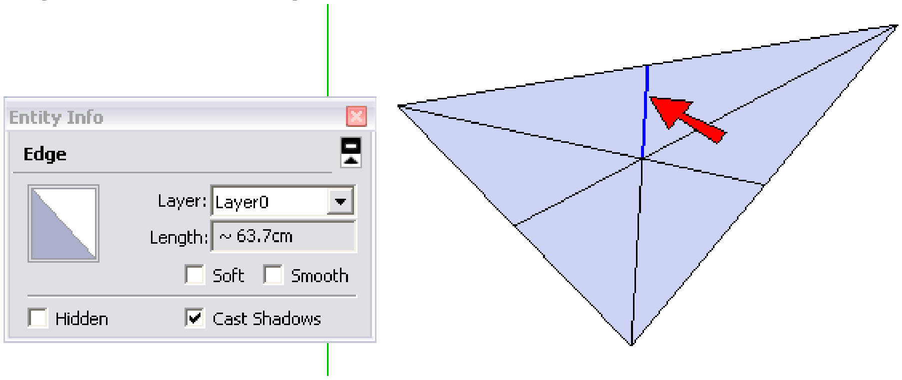


Notice the relationship? The longer segment is exactly double the shorter one.

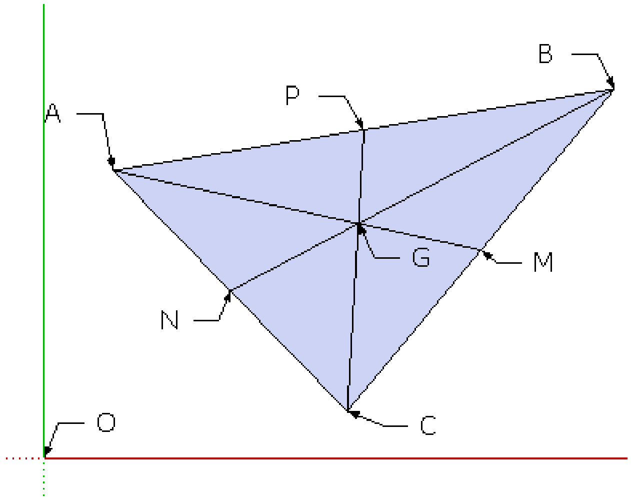
3. Let's check that this relationship applies to the other medians. The segment below is 127.5 cm long . . .



4. . . . and the other segment is half as long, 63.7 cm.



Teachers: If you have older students who know a bit about vector geometry, you can explore this 2:1 relationship further. Consider the geometry below, where O is the origin, M, N, and P are the edge midpoints, and G is the centroid.



1. These are the vector equations for the three midpoints:

OM = (1/2)OB+(1/2)OC
ON = (1/2)OA+(1/2)OC
OP =(1/2)OA+(1/2)OB

2. And here are the vector equations for the three medians:

AM = OM – OA
BN = ON – OB
CP = OP – OC

3. Combining the two sets of equations, we get:

AM = (1/2)OB+(1/2)OC - OA
BN = (1/2)OA+(1/2)OC - OB
CP = (1/2)OA+(1/2)OB - OC

4. Now we'll use vectors to get from the corner points to the centroid (G). First, start at A and then add on a multiple of AM. Expressed in vector form:

OG = OA + q AM (q is the multiplier: the ratio between AG and AM)

Similarly:

OG = OB + r BN
OG= OC + s CP

5. Now we can modify the equations in step 3 above, carrying the q, r, and s multipliers through both sides of the equations:

q AM = (q/2)OB + (q/2)OC – qOA
r BN = (r/2)OA + (r/2)OC – rOB
s CP = (s/2)OA + (s/2)OB – sOC

6. Substituting the equations from step 5 into the equations from step 4, we get:

OG = OA + (q/2)OB + (q/2)OC – qOA
OG = OB + (r/2)OA + (r/2)OC – rOB
OG = OC + (s/2)OA + (s/2)OB – sOC

7. We can restate the equations from step 6 as:

OG = (1-q) OA + (q/2)OB + (q/2)OC
OG = (r/2)OA + (1-r) OB + (r/2) OC
OG = (s/2) OA + (s/2) OB + (1-s)OC

8. Since all the coefficients of OA in step 7 have to be equal, we have:

1 – q = r/2 = s/2, which means that r = s

9. Since all the coefficients of OB in step 7 have to be equal, we have:

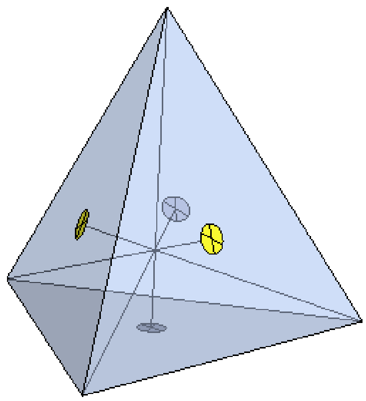
q/2 = 1 – r = s/2, which means that q = s

10. So we now know that q = r = s.

11. And since 1- q = r/ 2, we can substitute and get 1- r = r/2. Solving this we get that r = 2/3 = s = q.

This proves that the three medians meet at point G which is 2/3 the distance from a vertex to the midpoint of the opposite side.

Bonus: We also get the neat formula OG = (1/3)OA + (1/3)OB + (1/3) OC, by substituting q = 2/3 into OG = (1-q) OA + (q/2)OB + (q/2)OC. This confirms that the centroid location is the average of the three corner coordinates.

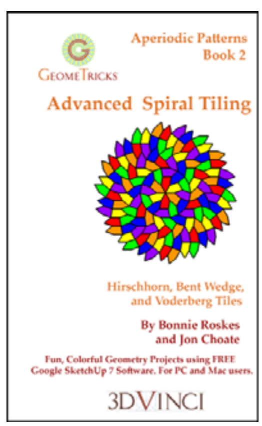


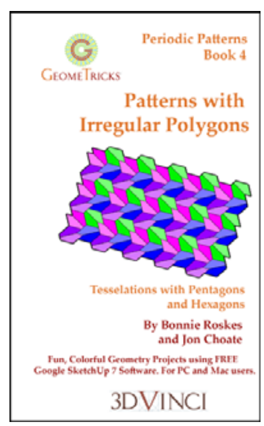
If you've got some SketchUp skills in 3D, you can extend this project's concepts with a tetrahedron. Again, all four medians meet at a single point, and the centroid divides each median by a 3:1 ratio.

The tetrahedron's vector proof is similar to the one for a triangle. I will add include this in a future Math Forum project!

Want More?

If you like geometric projects that use a little algebra, here are the GeomeTricks books that will make you grab your pencil and paper:

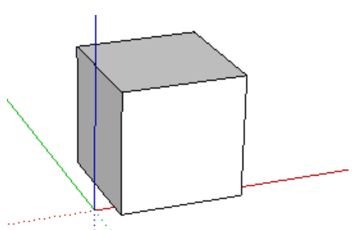


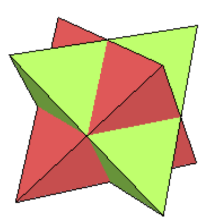


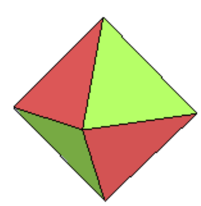
All books are available in print and as printable PDF. For details on GeomeTricks, go to http://www.3dvinci.net/ccp0-catshow/GM.html.

You can also sign up for our SketchUp Project of the Month subscription. Each month you will receive THREE FUN PROJECTS (one in math, two in 3D design) that can be used in K-12 classes. Details at http://www.3dvinci.net/ccp0-prodshow/POM.html.

For subscribers, the math project for April is about the relationships between cubes, tetrahedrons, and octahedrons.







The above project also has an accompanying video: http://3dvinci.blogspot.com/2010/04/new-geometry-video-cube-tetrahedron.html.