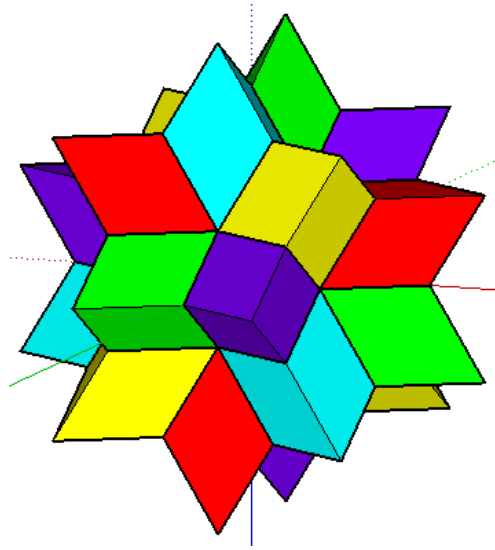


Rhombic Hexacontahedron in Google SketchUp

Check out this cool-looking shape:



You can read more about it here: <http://mathworld.wolfram.com/RhombicHexecontahedron.html>. It looks sort of complicated, and I'll admit it takes a number of steps to complete it. But once you're done, you'll have earned the right to proudly show it off!

This is one of the more complicated projects I've posted to the Math Forum, and it would be helpful for you to have a good knowledge of the **Rotate** tool, as well as familiarity with groups and components. But if you're a SketchUp newbie, just follow the steps closely and you should be just fine.

This project will work in any version of SketchUp, including Version 8.

Teacher Note: All text that appears in **red** is for the teacher version only, and does not appear in the student version.

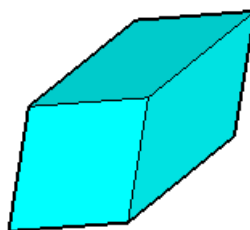
For this project, it helps to have some basic knowledge of Google SketchUp (though detailed instructions are provided). In particular, it's important to know how to zoom, orbit, and pan the view. If you need more information on how to get started, and a description of some basic tools, please read 3DVinci's Getting Started Guide (PDF).

PC users: go to http://www.3dvinci.net/SketchUp_Intro_PC.pdf.

Mac users: go to http://www.3dvinci.net/SketchUp_Intro_MAC.pdf.

Step 1: Create a Rhombohedron

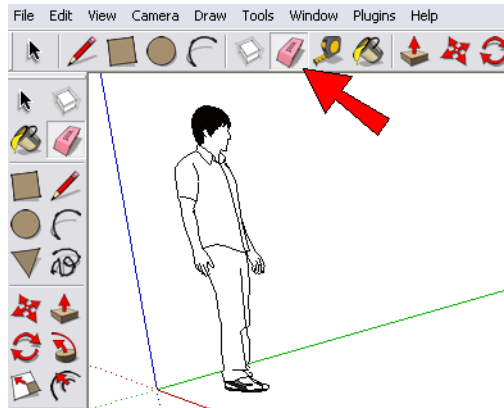
The rhombic hexacontahedron is composed of 60 faces, each of which is an identical rhombus. There are a few ways to create this object, but the way I've chosen to show here starts with one rhombohedron. A rhombohedron is like a cube, except that all of its six faces are identical rhombi rather than squares.



The final model for this project is composed of 20 rhombohedra, so we'll have to do a bit of copying.

For each one of these rhombohedra, three of its six faces are visible and three are "buried inside" the object. So in the end, you'll see $3 \times 20 = 60$ faces.

1. Open Google SketchUp. If your file contains a person standing on the ground near the origin, click the **Eraser** tool and erase him or her.



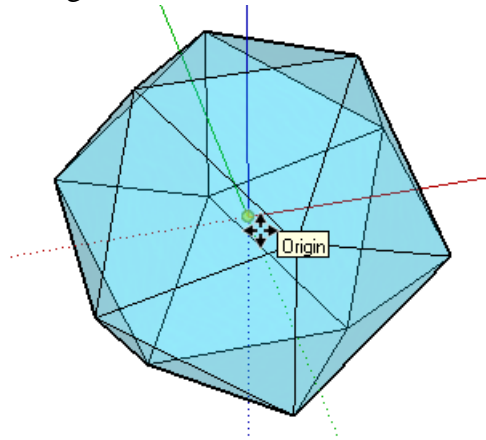
2. We'll start by downloading a model of an icosahedron, on which we'll build the new model. Click the **Get Models** tool.



3. In the search field, enter "centered icosahedron roskes." For the model shown below (by me), click "Download Model."

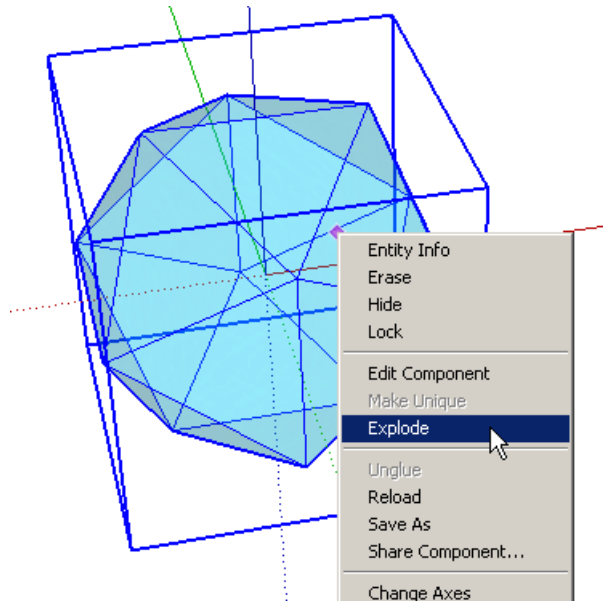


4. Place the icosahedron exactly at the origin. (The faces are translucent so that you can see through to find the origin.) Now each vertex of each edge is the same distance from the origin.

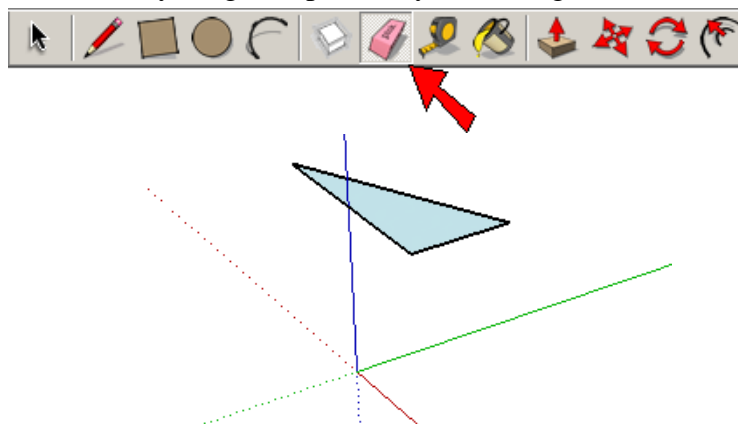


Teacher Note: We're using a pre-made icosahedron, but it's not hard to create one yourself. For one explanation, check out 3DVinci's Teacher Guide (<http://3dvinci.net/teacherguide>), and go to the project for Math 10-12.

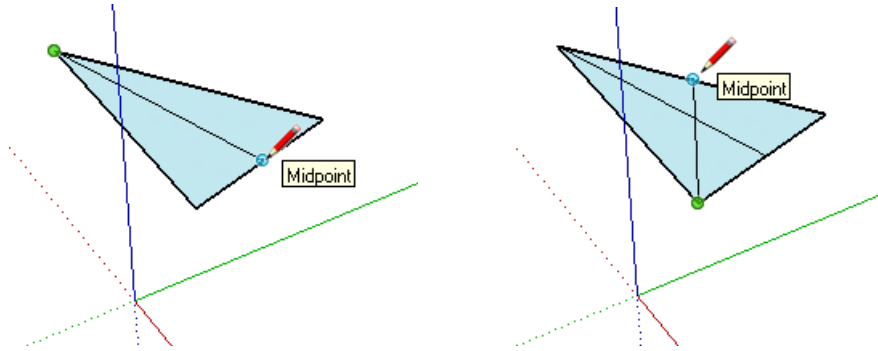
5. Because we only need a few edges of the icosahedron, right-click on it and choose **Explode**. Now the icosahedron can be edited.



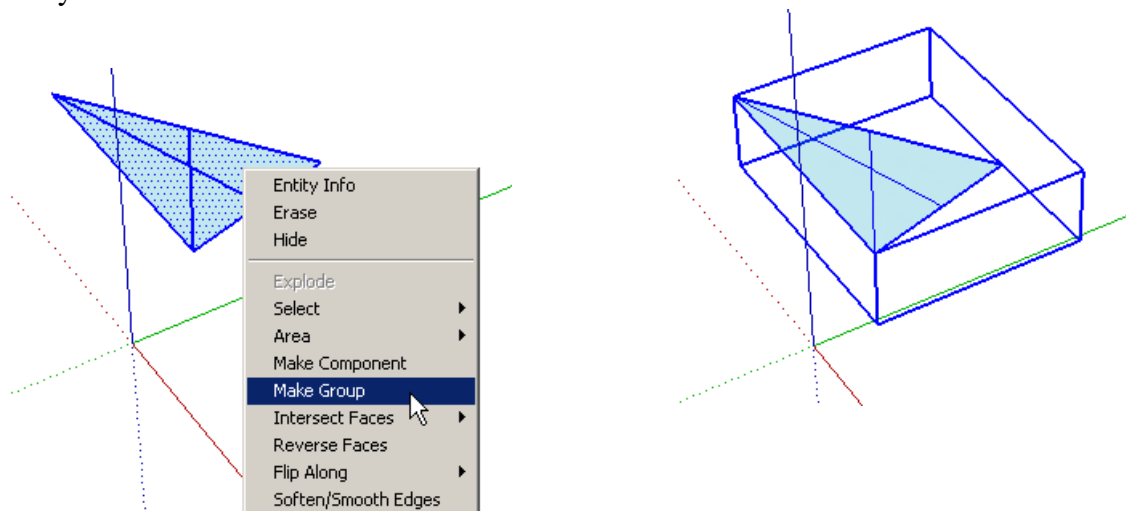
6. Activate the **Eraser** and erase everything except for any one triangle.



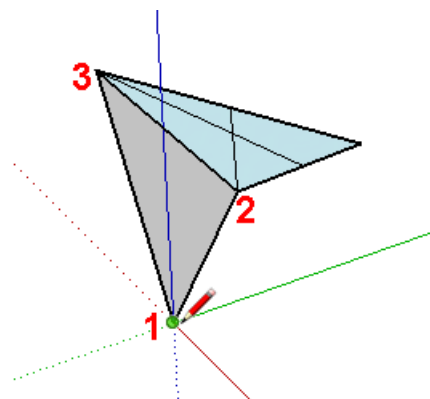
7. We now have to mark the center of this triangle. Activate the **Line** tool and draw two lines as shown below, each from one corner of the triangle to the midpoint of the opposite edge. Where these two lines intersect is the center of the triangle.



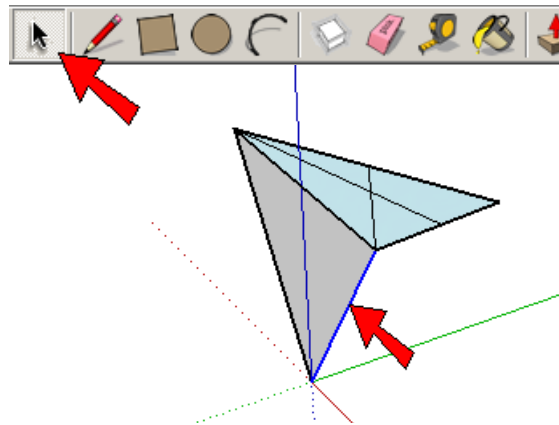
8. So that we can easily erase this triangle later, select the entire thing (press Ctrl + A on PC, Cmd + A on Mac), right-click on the triangle, and choose **Make Group**. Now the triangle and its extra lines are a single object, which can easily be erased later.



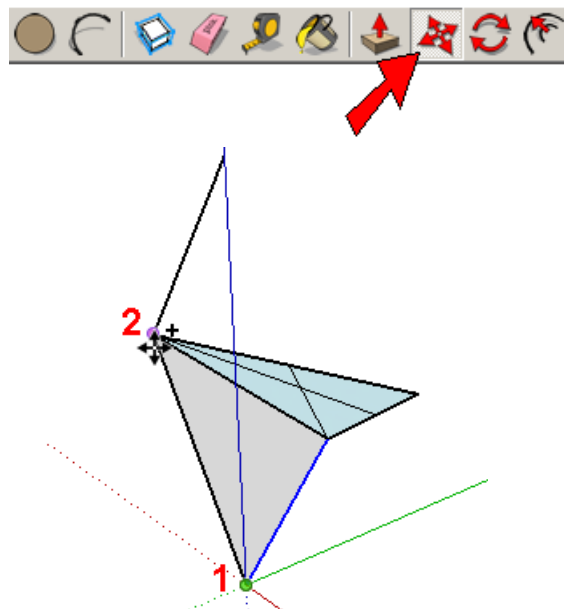
9. This triangle is all we need to create the first rhombohedron. To start the rhombohedron, use the **Line** tool to draw a triangle as shown below, with one corner at the origin (Point 1), and sharing the other two corners (Points 2 and 3) with the triangle inside the group. You'll have to click all three points to complete the new triangle.



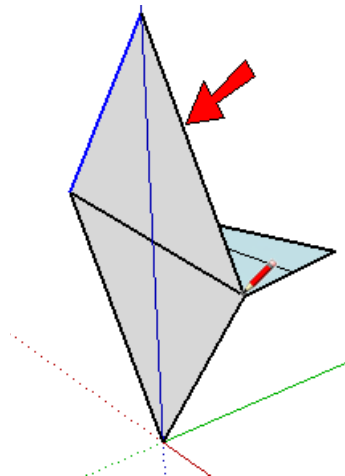
10. We'll use this triangle to create the first rhombus. Activate the **Select** tool and click one of the triangle edges that meets the origin.



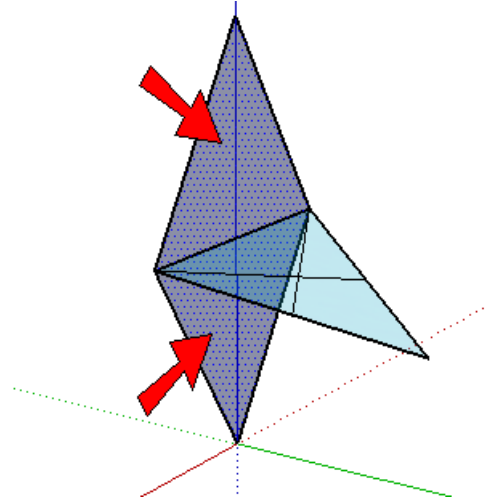
11. To copy this edge, activate the **Move** tool and press the Ctrl key on PC, Option key on Mac. (You don't have to keep this key pressed, just tap it once. You should see a "plus" sign next to your cursor.) Then click Point 1 and Point 2 below, to copy the edge.



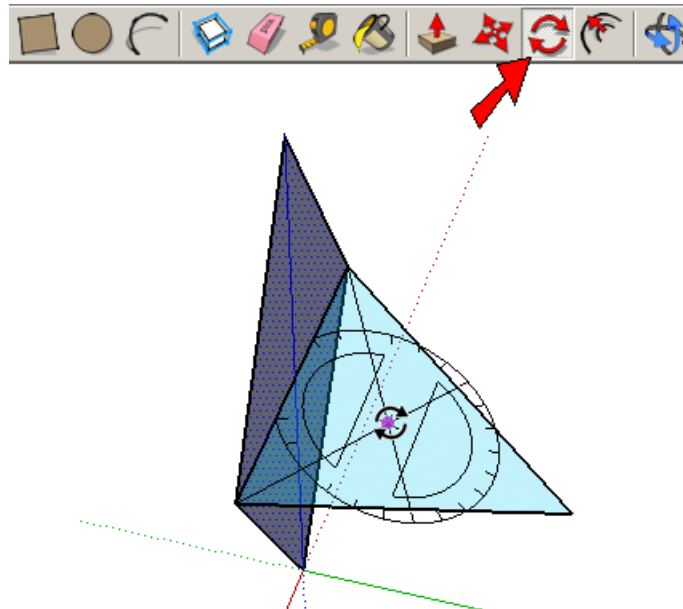
12. Use the **Line** tool to draw the missing edge, to complete the rhombus.



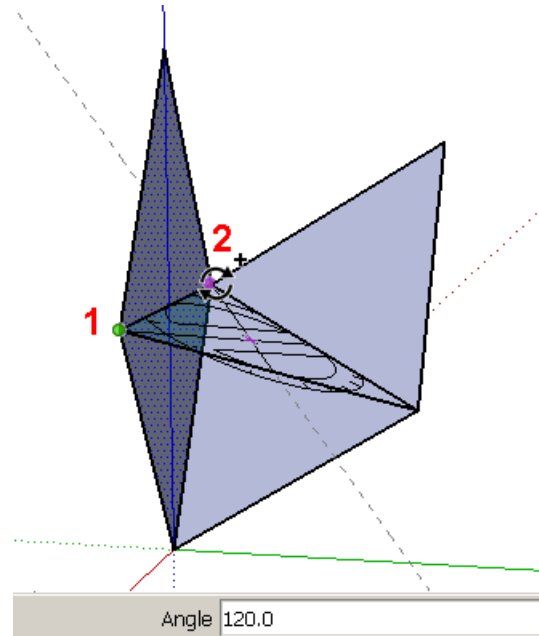
13. Now this rhombus will be rotate-copied. Activate **Select** again, press and hold the Shift key to select multiple objects, and select both triangles that comprise the rhombus.



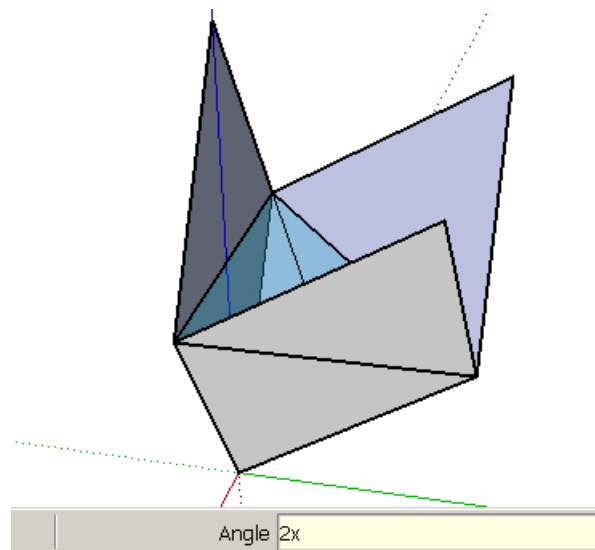
14. Activate the **Rotate** tool, and place the protractor at the center point of the triangle inside the group.



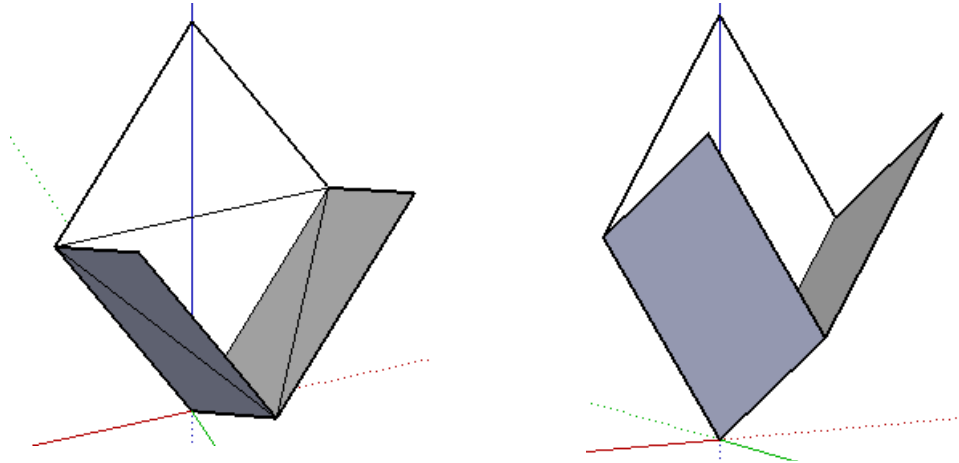
15. Press Ctrl or Option again to make a copy, then click any two corner points of the triangle inside the group. Look at the **Angle** field at the lower right corner, and you should see 120 degrees.



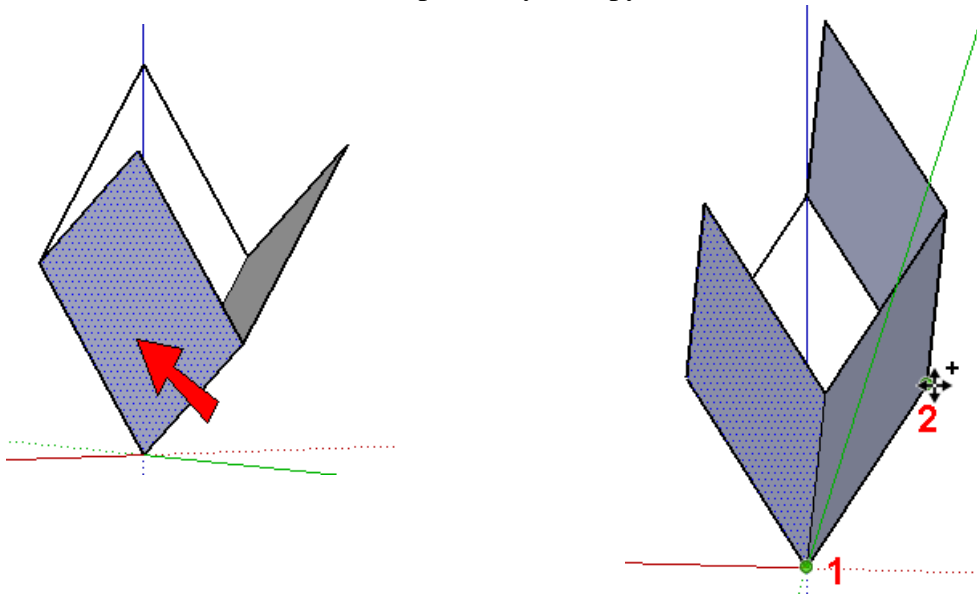
16. Just after creating that copy, type 2x, which now appears in the **Angle** field. (Don't click in this field, just type and the numbers will appear.) Press Enter, and now there are two copies instead of one, for a total of three rhombi.



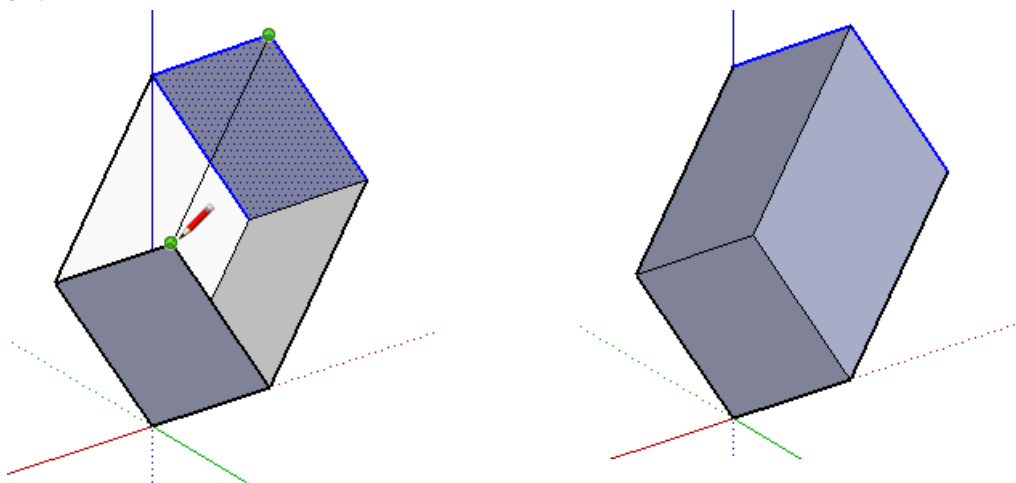
17. The triangle inside the group is no longer needed, so use the **Eraser** to erase it. And while you're at it, erase all of the edges that divide the three rhombi.



18. We have three faces of the rhombohedron, but we need three more. Start by selecting the rhombus closest to you. Then use the **Move** tool with the Ctrl / Option key to copy this face from Point 1 to Point 2.



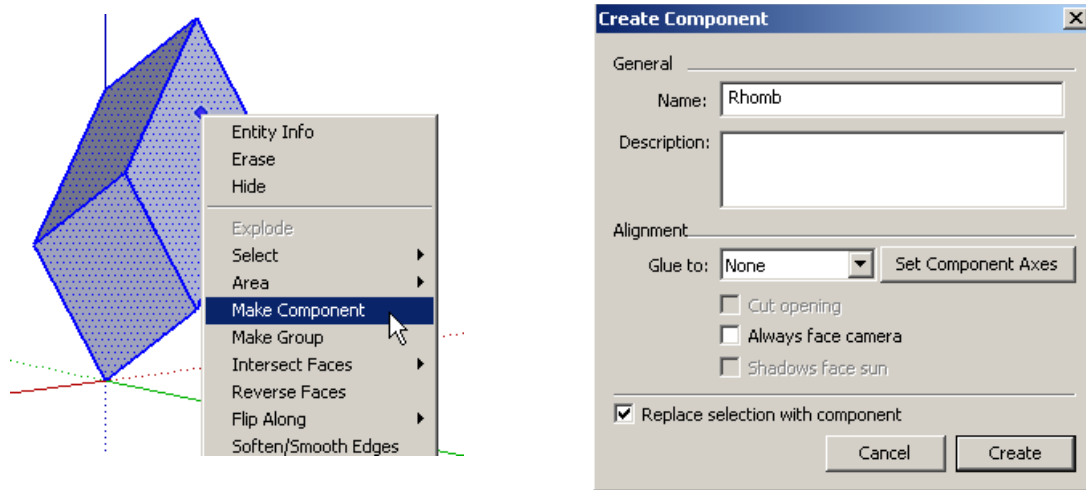
19. Then use the **Line** tool to draw the edge that will complete the two remaining faces. Voila - the first rhombohedron!



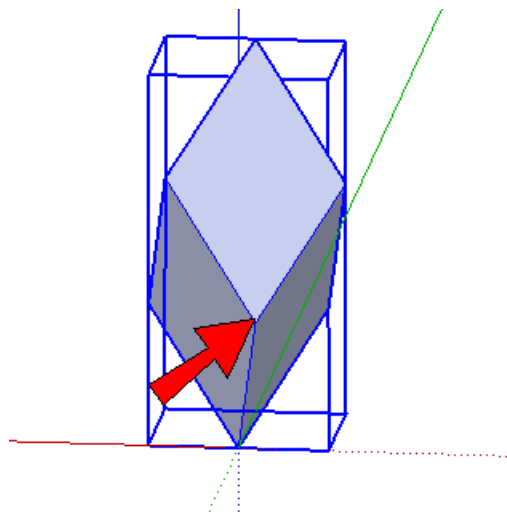
Step 2: Make More Rhombohedra

Now comes the part where we use the **Rotate** tool several times. We're also going to bring in some color - painting each rhombohedron so that around each meeting point, no rhombohedron has a repeated color. Since each vertex is surrounded by five rhombohedra, we'll need five colors.

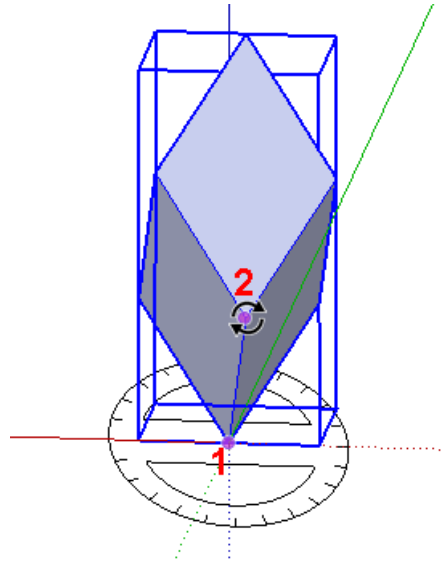
1. Whenever you have an object that will be repeated, it should be made into a component. So select the entire rhombohedron, right-click on any face, and choose **Make Component**. Assign it any name, make sure **Replace selection** is checked, and click **Create**. Once the component is created, leave it selected (highlighted in the blue box).



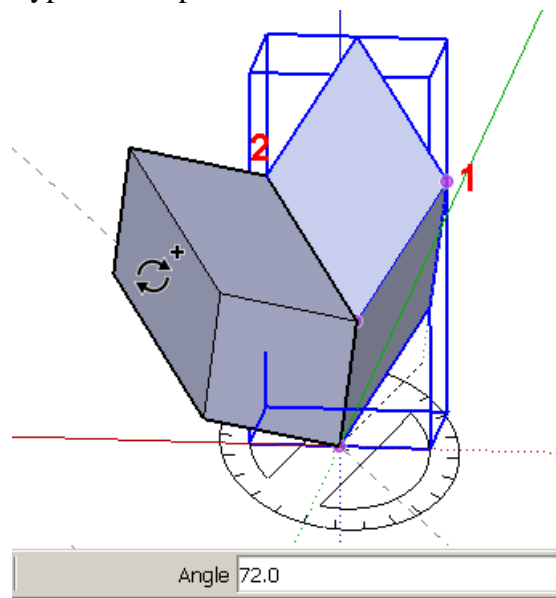
The corner indicated below is the first vertex we'll use - four more rhombohedra will be copied around this point. Three edges of your rhombohedron should have endpoints at the origin; your vertex can be at the *non-origin* endpoint of any of these edges.



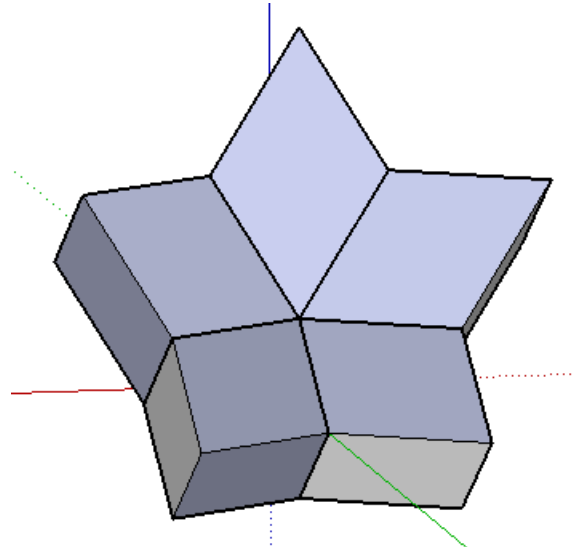
2. Activate the **Rotate** tool. Instead of clicking to place the protractor at a point, we'll be *dragging* the mouse in order to define a rotation axis. This simply means to click Point 1 (the origin), drag the cursor to Point 2 (the vertex), and release the mouse button.



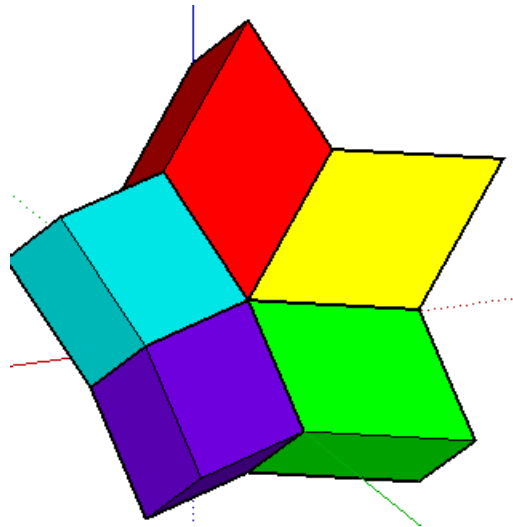
3. Press Ctrl / Option, and click Points 1 and 2, to create the first copy. The rotation angle should be 72 (which equals $360 / 5$). If you can't click Point 2 (sometimes SketchUp is funny about "finding" points), you can place the copy anywhere, then type 72 and press Enter.



4. Type 4x and press Enter. Now there are four copies, for five total rhombohedra.

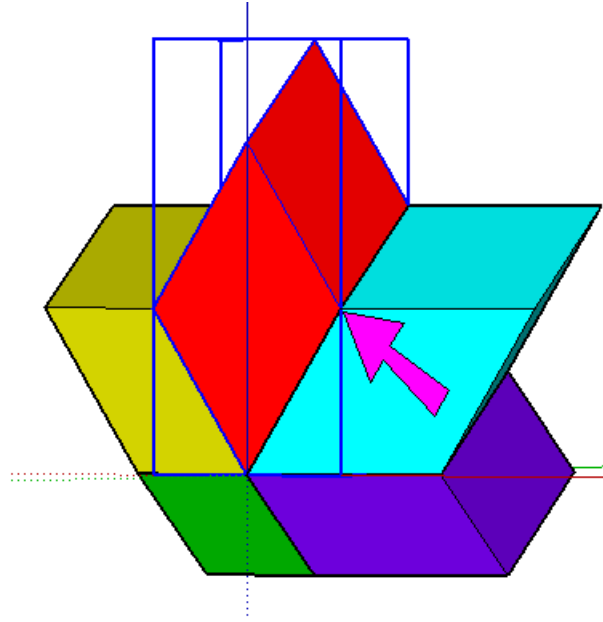


5. Click the **Paint Bucket** to open the **Materials** window (**Colors** window on the Mac) and find the folder with solid colors. Paint the rhombohedra *exactly* like this - following the colors I've specified here will make the steps easier to follow.

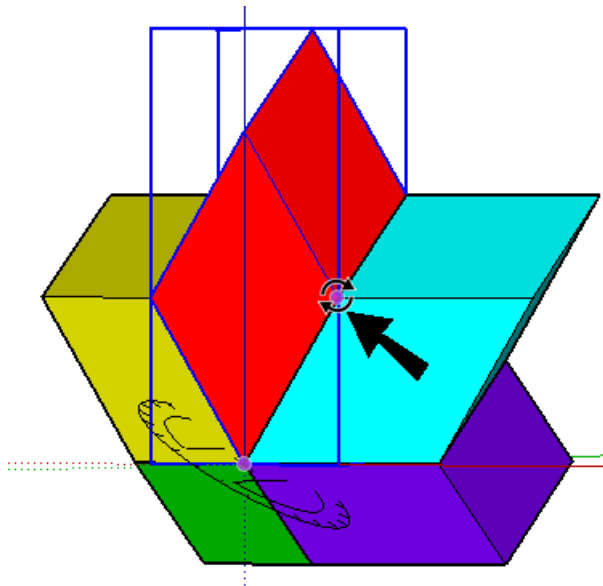


You don't have to edit the components to paint them, just choose a color and click the rhombohedron to paint the whole component the selected color.

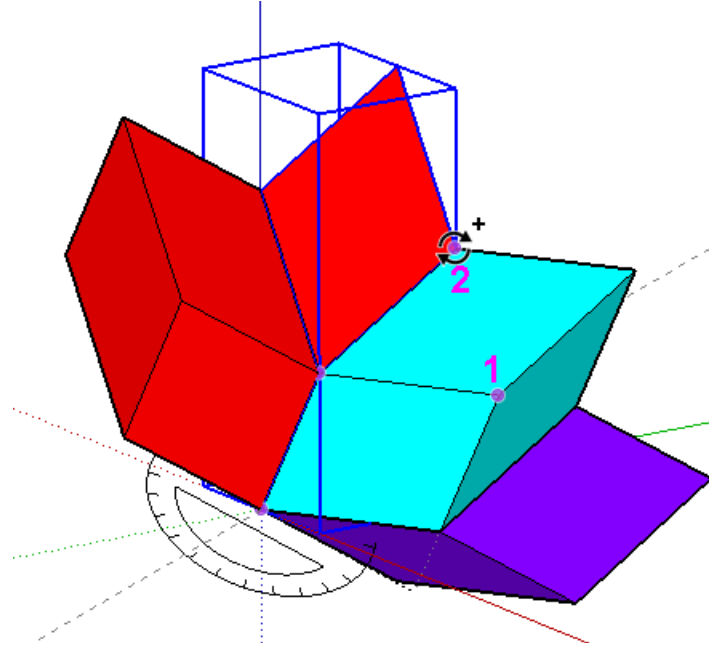
6. Now orbit to face the other side of the five rhombohedra, with the red one “pointing” toward the top. Select the red one.



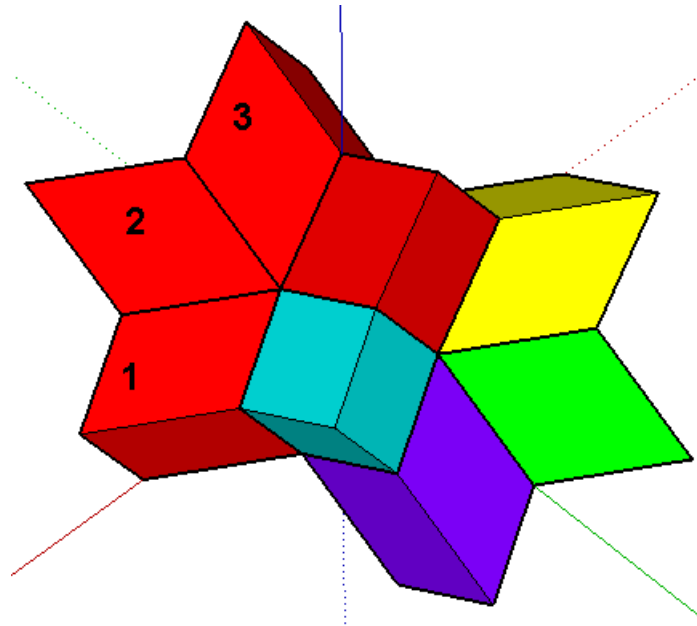
7. Activate **Rotate**, and drag the cursor from the origin to the point indicated below, between the red and cyan components. This point is a vertex currently surrounded by two rhombohedra: red and cyan. We need three more rhombohedra around this vertex.



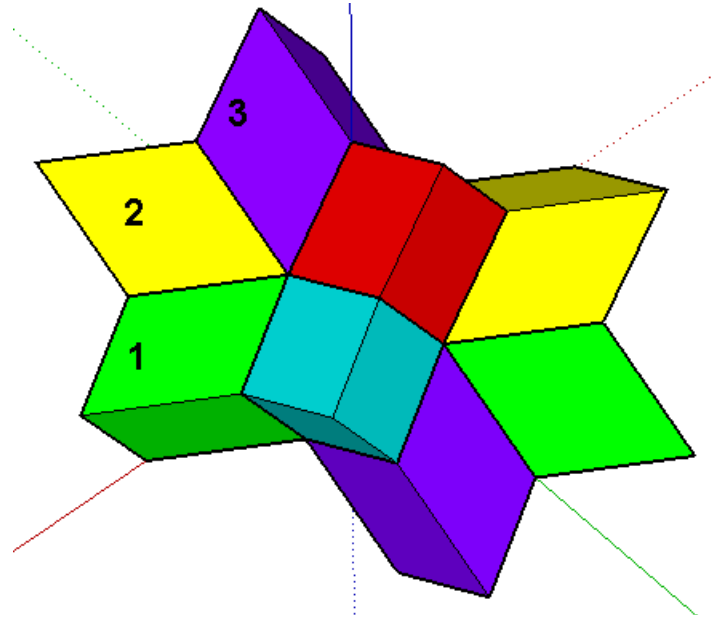
8. Press Ctrl / Option and make the first copy by clicking Points 1 and 2 of the cyan component. Again, the angle should be 72 degrees.



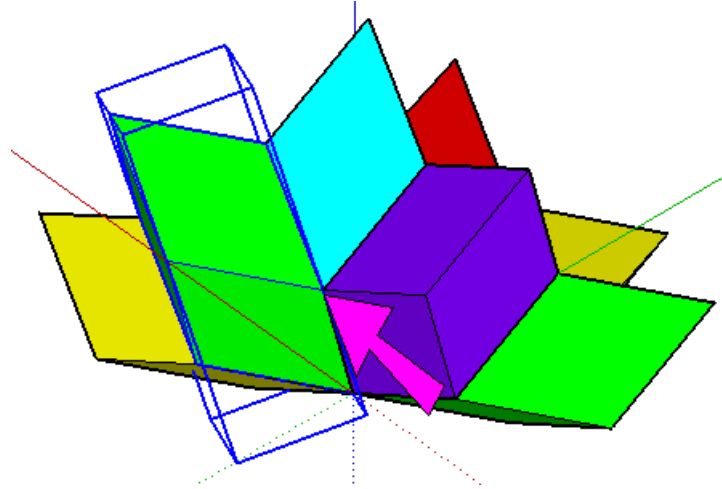
9. Enter 3x to make three more copies; now five rhombohedron surround this vertex. Orbit to a view exactly like this. There are three too many red rhombohedra - 1, 2, and 3 below.



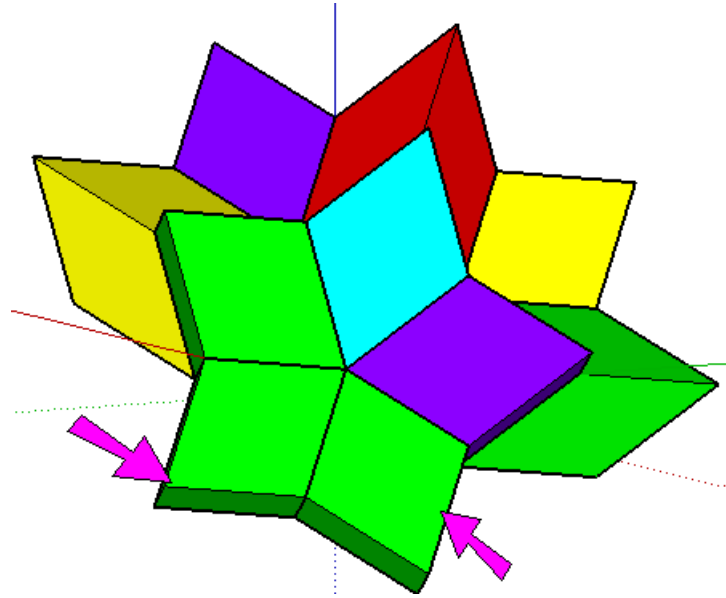
10. Paint these three rhombohedra exactly as shown below. This order ensures that we can keep going with our pattern of no repeated colors around any vertex.



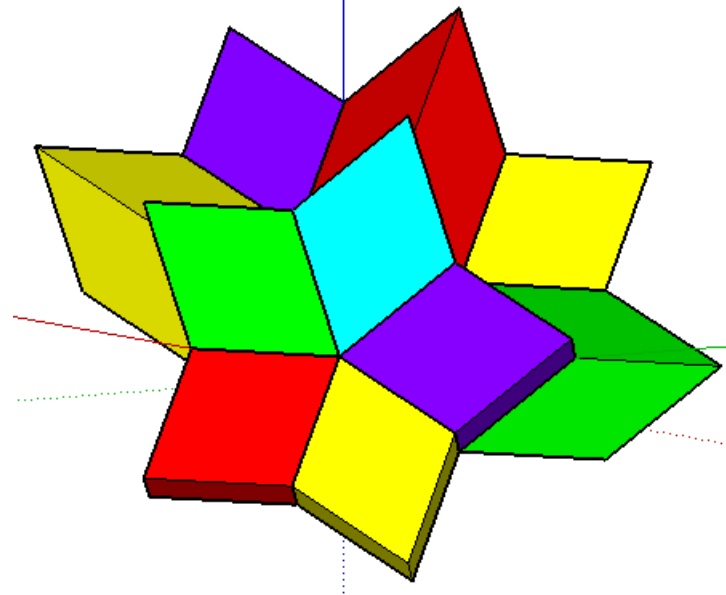
11. The rest of the copies can be done the same way, and now that the first few colors are established, figuring out how to paint the rest should be clear. To get you started on the next copy, find the green rhombohedron that's next to the cyan one, and select it. The vertex we'll copy around is indicated below. There are already three rhombohedra around this vertex, so only two more are needed.



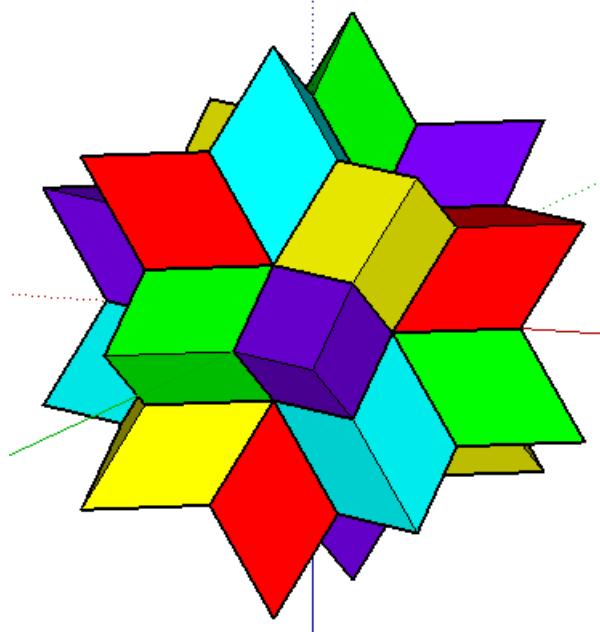
12. Make the two copies, this time entering 2x for the number of copies.



13. By paying attention to the colors already in use, there is only one way to paint these extra green ones. The two missing colors are red and yellow, and to avoid repeats, the two colors have to be applied like this:



14. Now that you know the steps, keep going around a few more vertex points until all 20 rhombohedra are filled in.



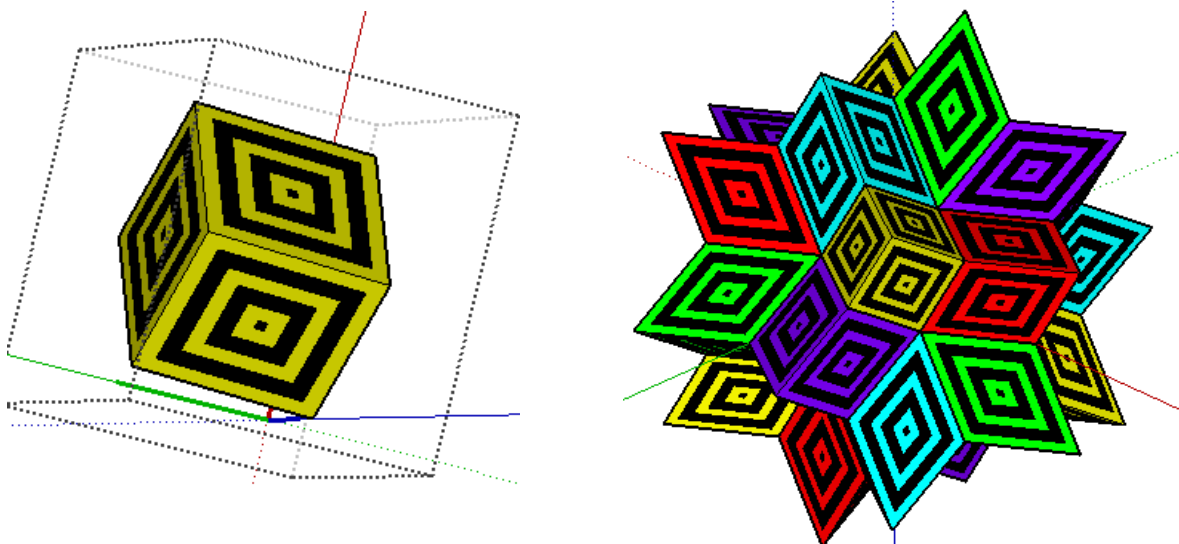
Step 3: Make Changes

Each rhombohedron is a component, which means any change you make to one gets applied to all of the others. So now you can get creative and see what you can come up with by editing any of the components.

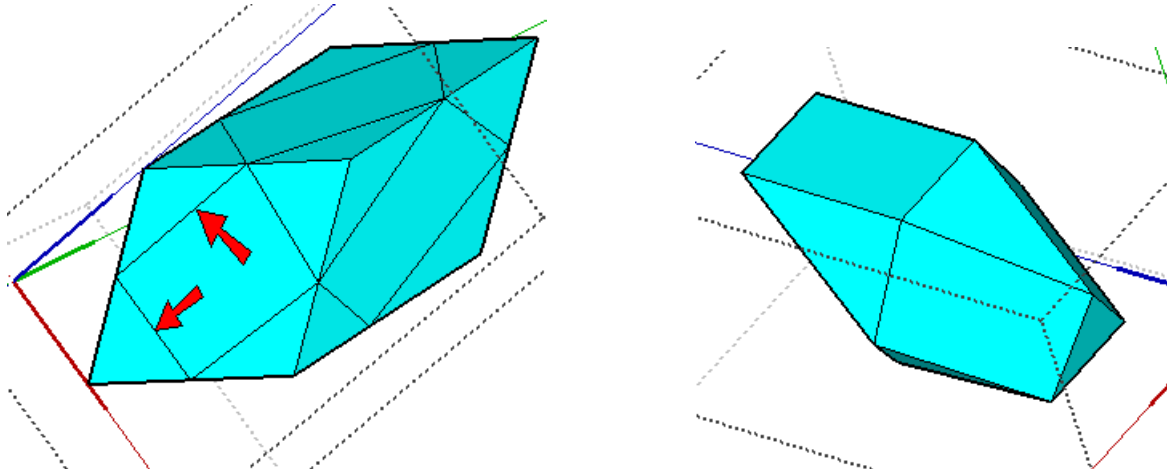
*You can edit a component by right-clicking on it and choosing **Edit Component**. If you want to “blank” the rest of the model while you’re doing your editing, open **Window / Model Info** to the **Components** page, and check both **Hide** boxes. When you’re finished with a component, right-click in blank space and choose **Close Component**.*

Here are some ideas I came up with:

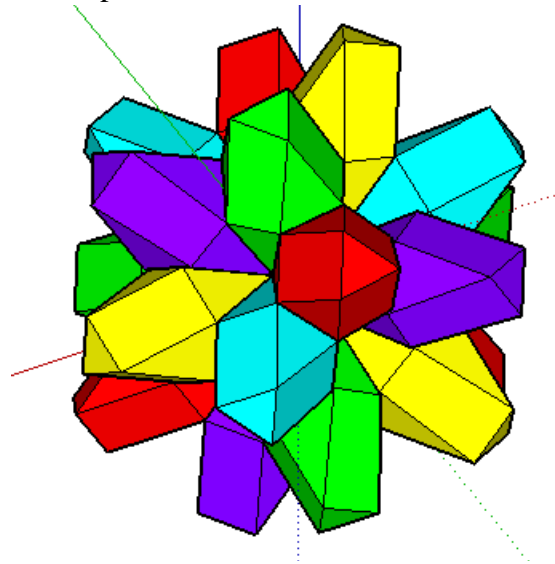
Add zebra stripes to the faces of the rhombohedron, using the **Offset** tool. (After you make the first offset, you can double-click to make subsequent offsets that use the same distance.) Paint alternating stripes black. When you close the component, there are stripes everywhere.



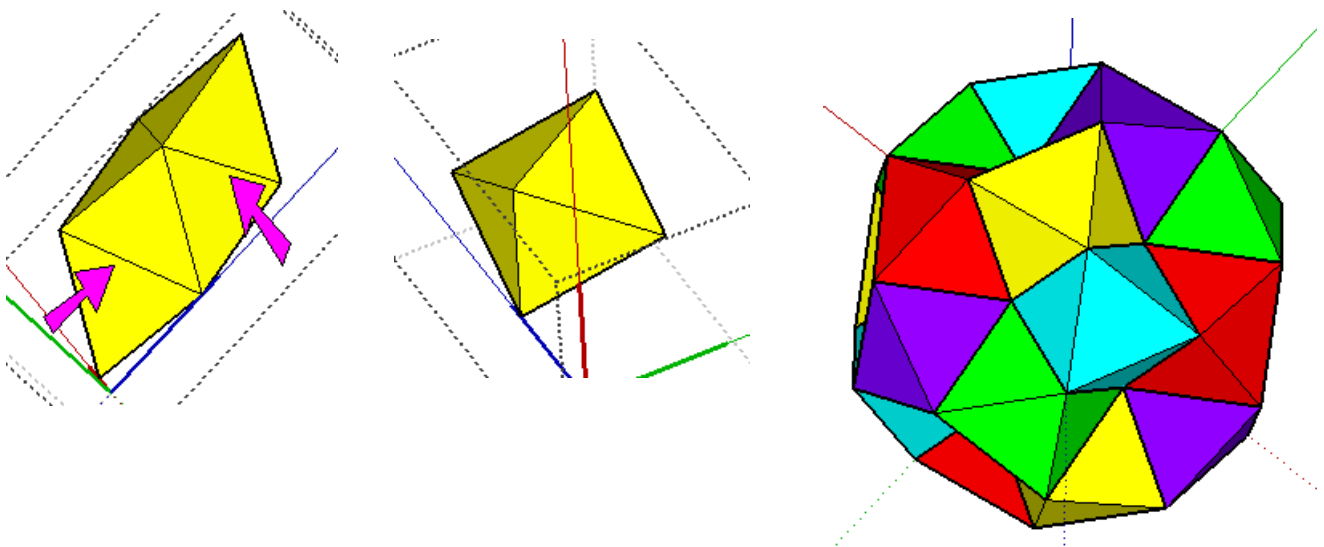
Another idea: on any rhombohedron component, add lines on each face from midpoints to midpoints. Then use the **Eraser** to “shave off” the extra pyramids at the corners.



Here’s what you get after closing the component:



Last idea: on any rhombohedron, draw a line on each face that divides the rhombic face in half. Then shave off the two pyramids at the ends, until you’re left with an octahedron: a solid with eight identical triangles. Pretty neat result!



Teachers, Want More?

For more projects on 3D geometry, you'll love 3DVinci's GeomeTricks books on Platonic, Archimedean, and related solids:



All books are available in print and as printable PDF. For details on GeomeTricks, go to <http://www.3dvinci.net/ccp0-catshow/GM.html>.

If you sign up for our [SketchUp Project of the Month](http://www.3dvinci.net/ccp0-prodshow/POM.html) subscription, each month you will receive THREE fun projects (one in math, two in 3D design) that can be used in K-12 classes. Details at <http://www.3dvinci.net/ccp0-prodshow/POM.html>.

February's project shows how to make this Mandala pattern:

