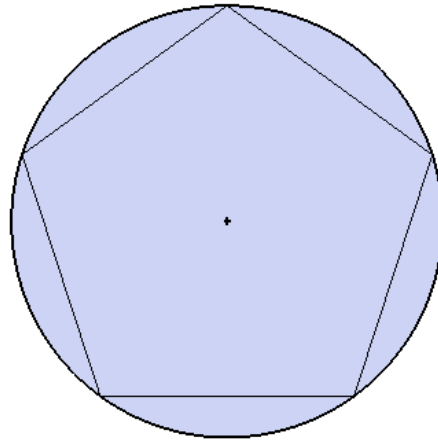


Constructing a Pentagon from a Circle, in Google SketchUp

There is a very nice animation of how a pentagon can be created from a circle on this Wikipedia page:

http://en.wikipedia.org/wiki/File:Pentagon_construct.gif

This project shows how to create this construction in Google SketchUp.



All you need to complete this project is a few SketchUp tools.

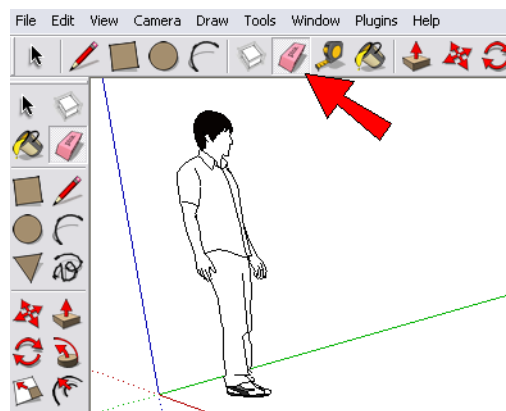
Teacher Note: All text that appears in **red** is for the teacher version only, and does not appear in the student version.

For this project, it helps to have some basic knowledge of Google SketchUp (though detailed instructions are provided). In particular, it's important to know how to zoom and pan the view. If you need more information on how to get started, and a description of some basic tools, please read 3DVinci's Getting Started Guide (PDF).

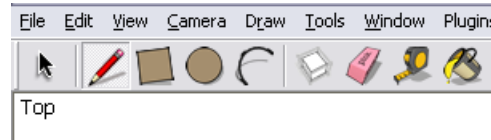
PC users: go to http://www.3dvinci.net/SketchUp_Intro_PC.pdf.

Mac users: go to http://www.3dvinci.net/SketchUp_Intro_MAC.pdf.

1. Open Google SketchUp. If your file contains a person standing on the ground near the origin, click the **Eraser** tool and erase him or her.



2. From the main menu, choose **Camera / Standard Views / Top**. Now you're looking down on the "ground," and the word **Top** appears in the top left corner of the SketchUp window.



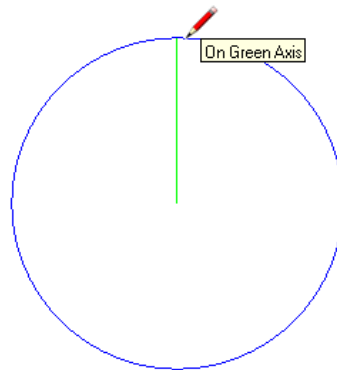
3. Activate the **Circle** tool.



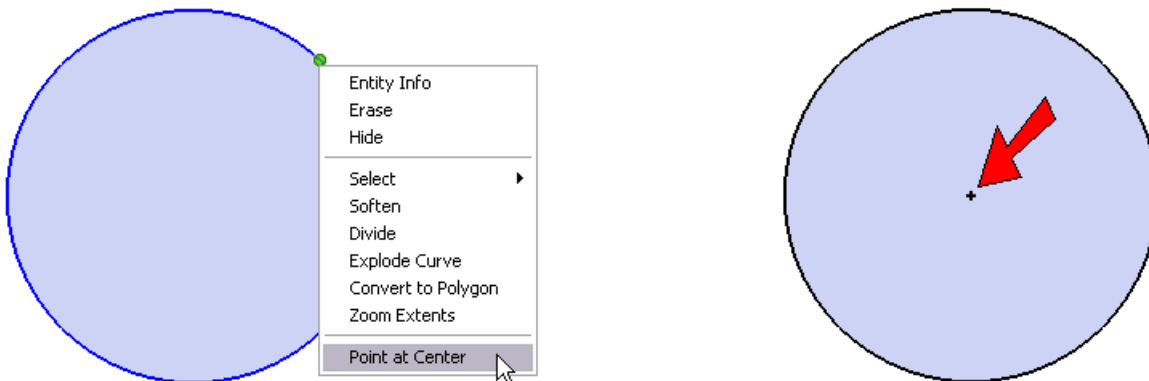
4. It's important to keep in mind that circles in SketchUp aren't really circles - they are actually segmented polygons. The default number of circle "sides" is 24, but to more closely approximate a circle, this number should be much higher. Type the number 240, which appears in the **Sides** field in the lower right corner of the SketchUp window. (Don't click in this field, just type and the number will appear.) Then press Enter.



5. To create the circle, click anywhere to place the center, then move the mouse in either the green direction (vertically) or the red direction (horizontally) and click to complete the circle.

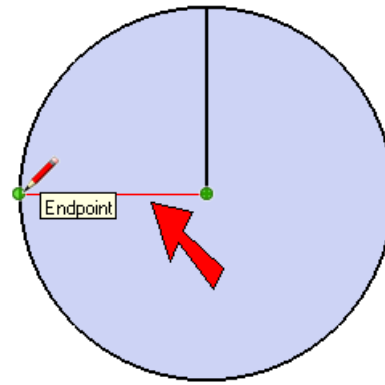
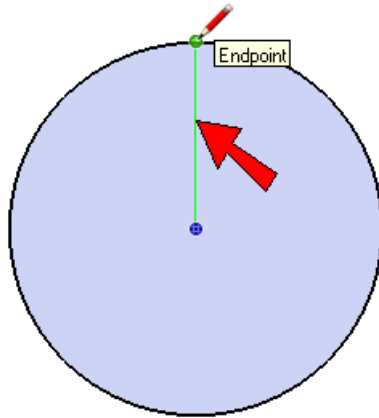


6. To mark the center of the circle, right-click on the circle's edge and choose **Point at Center**. A small dot appears at the center.

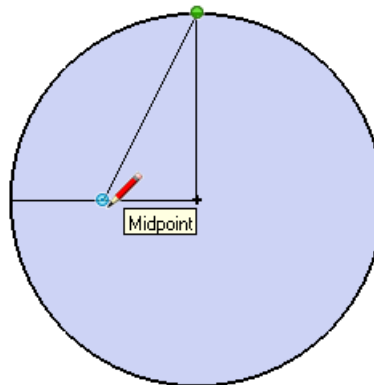


If the **Point at Center** option does not appear, open **Window / Preferences (PC)** or **SketchUp / Preferences (Mac)** to the **Extensions** page and check **Ruby Script Examples**.

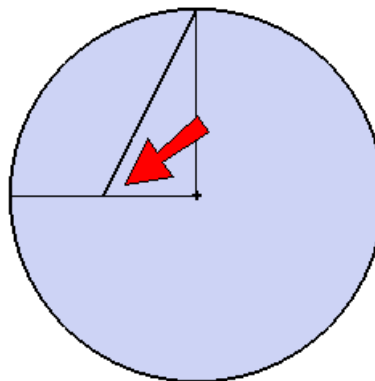
7. Activate the **Line** tool and draw a line starting at the center point and proceeding straight up, in the green direction, until you reach an endpoint along the circle. Then draw another line from the center proceeding to the left, in the red direction, until you reach an endpoint.



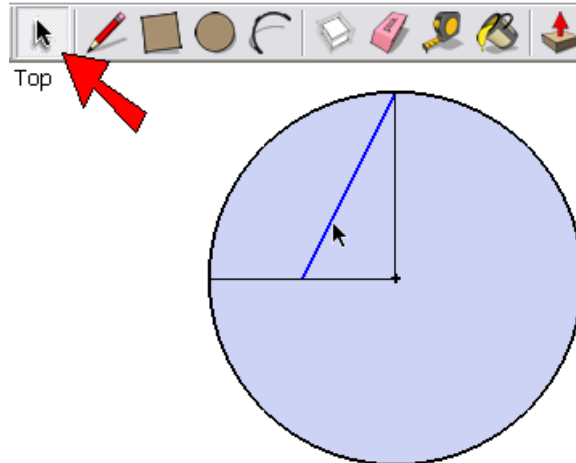
8. For the next line, start at the top of the vertical line, and end at the *midpoint* (look for the cyan midpoint indicator) of the horizontal line.



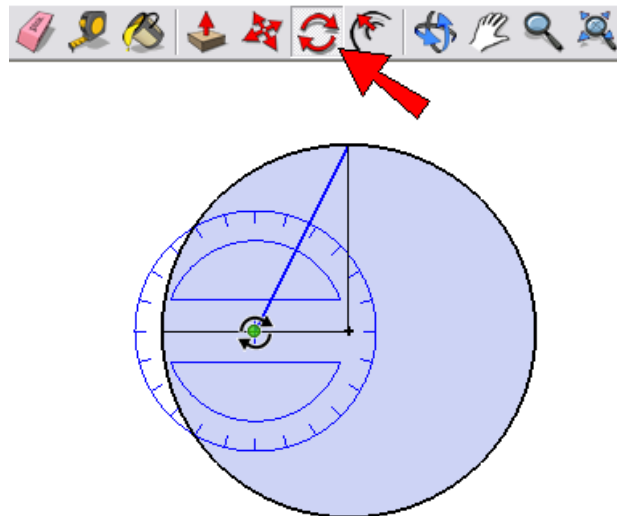
The next step is to add a line starting at the bottom of the diagonal line you just drew, which will bisect (divide in half) the angle indicated below. We could measure this angle and halve it, but there's a more exact way in SketchUp to bisect this angle.



9. We'll make a rotated copy of the diagonal line, but first we need to select what's going to be copied. So activate the **Select** tool, then click the diagonal line to select it.

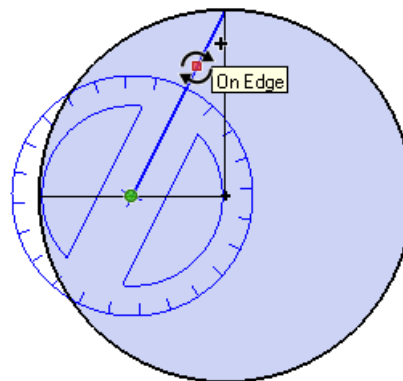


10. Activate the **Rotate** tool, and click to place the protractor at the bottom of the diagonal line.

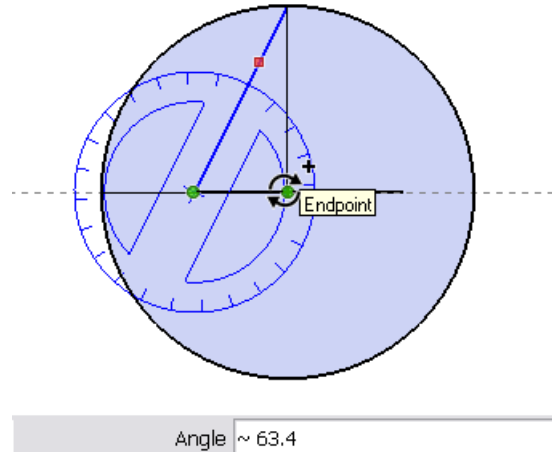


11. Press the Ctrl key (PC) or the Option key (Mac), so that a copy will be made. You don't have to keep this key pressed - just tap it once. You should now see a "plus" sign attached to your cursor.

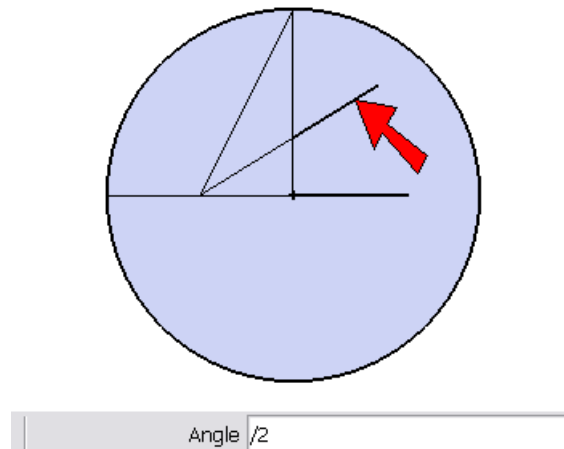
12. The next two clicks define the start and end angles. For the start angle, click anywhere along the diagonal line.



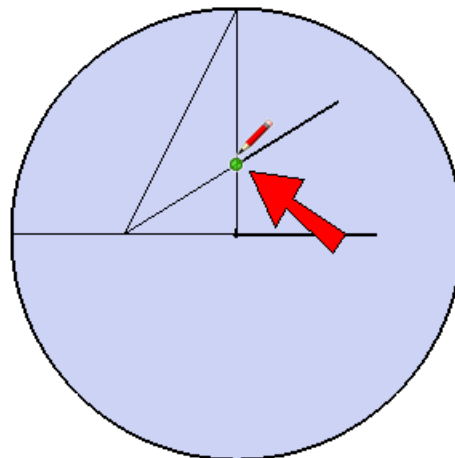
13. For the end angle, click anywhere on the right half of the horizontal line. Look at the **Angle** field, which lists a rather inexact ~63.4 degrees. Taking half of this angle would also be pretty inexact.



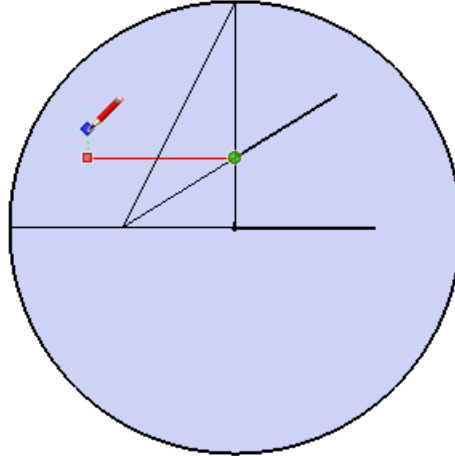
14. Because we want to bisect this angle, we want another copied line in the middle of the angle just measured. So before clicking anywhere, type $2/$ (don't forget the slash symbol for division) and press Enter. This creates the line indicated below, halfway between the diagonal and horizontal lines.



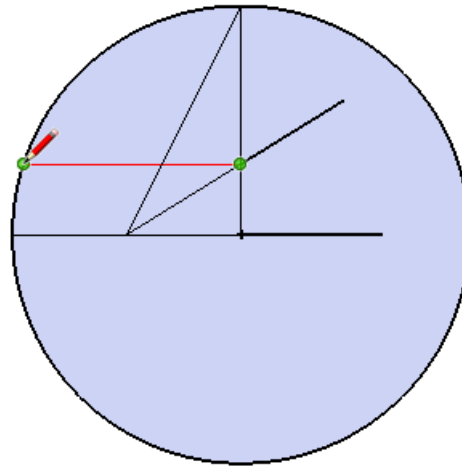
15. Activate **Line** and start the next line where the bisecting line meets the vertical line.



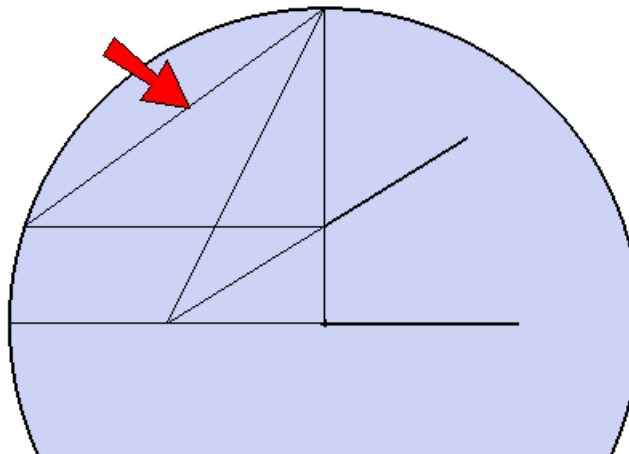
16. This line needs to be horizontal, so press the right arrow key to lock the line to the red direction. Now wherever you move the cursor, the line always stays red.



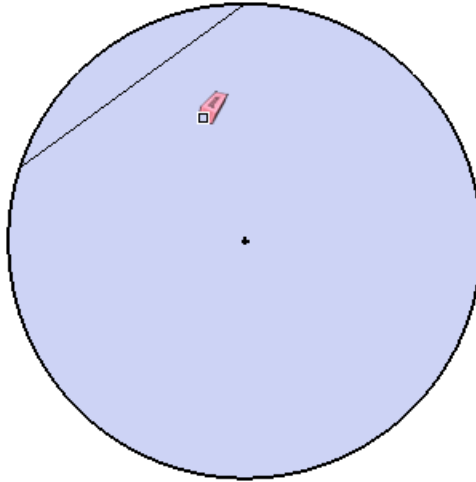
17. Click to end the line where it meets the circle. You may have to zoom in closely to click the exact endpoint where this line hits the circle.



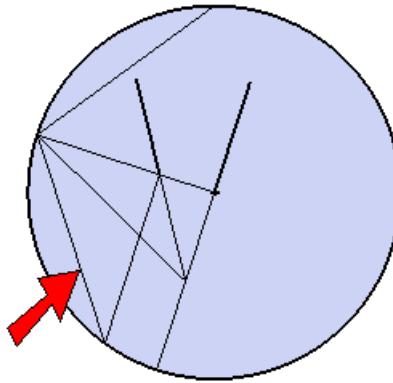
18. Now we have both endpoints of one of the pentagon edges - draw the line shown below.



19. Erase everything inside the circle except for the center point and the pentagon edge you just drew.

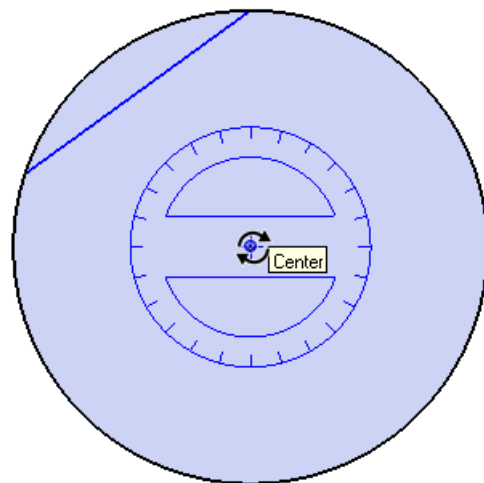
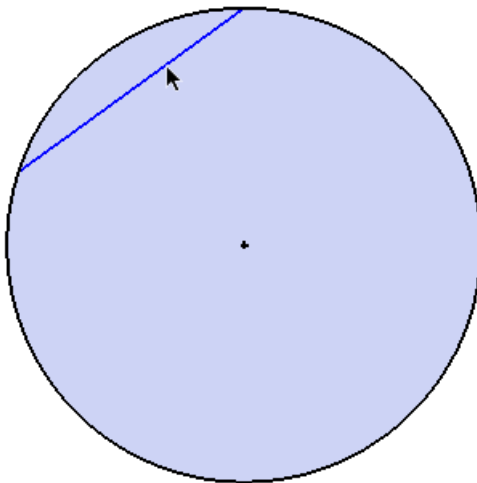


You could repeat this process of creating lines for the other four edges, since SketchUp enables you to easily create parallel and perpendicular lines. For example, the lines for the next edge would be set up like this:

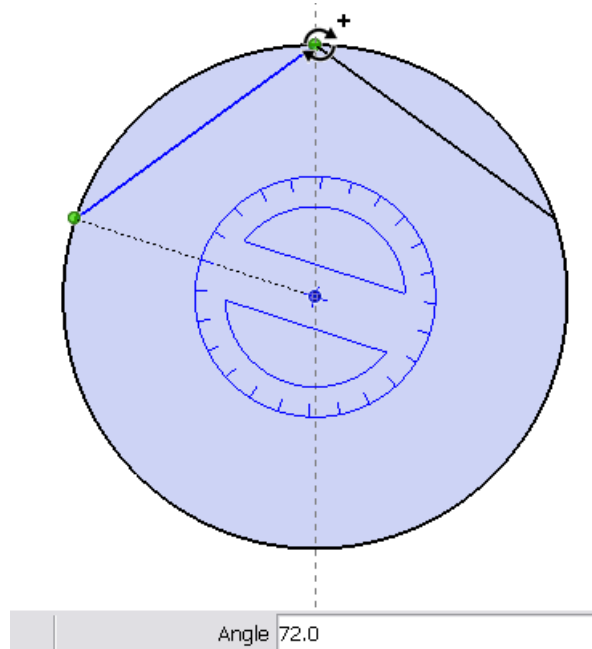


But since we already know how to use the **Rotate** tool to make copies, a much easier way to finish the pentagon is to just rotate-copy the one pentagon edge we already have.

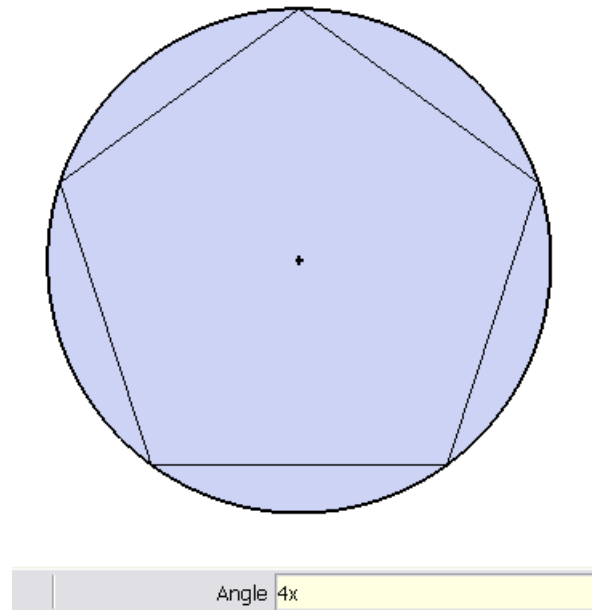
20. Select the edge, activate **Rotate**, and place the protractor at the center of the circle.



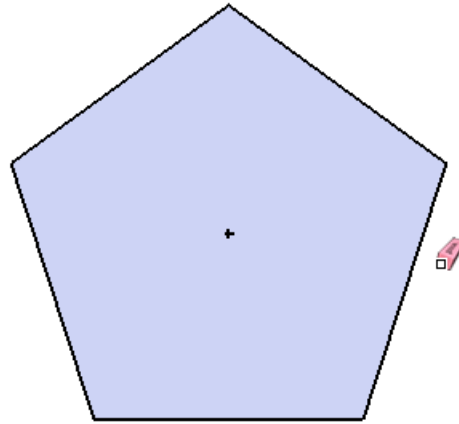
21. Press Ctrl or Option to make a copy, and for the start and end angles, click both endpoints of the pentagon edge, in either order. Look in the **Angle** field - the rotation angle should be 72 degrees ($5 * 72 = 360$).



22. Immediately after the copy is made, type 4x so that four copies will be created instead of just one. If you've done it correctly, you'll have five pentagon edges that loop back to the start point.



23. Use the **Eraser** to remove the edges of the circle, and you're done!



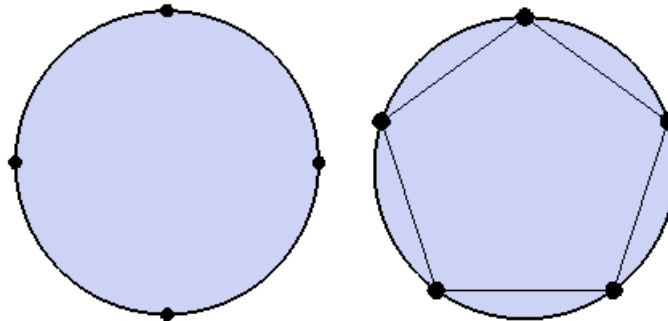
Start with a Different Circle

To create the pentagon, we started with a circle that had 240 edges. But with so many edges, it's a bit hard to find each tiny segment's endpoints, making it necessary to zoom in closely when creating some of the construction edges.

This pentagon construction can be done with a circle that has many fewer edges. What is the minimum number of edges you'd need to use?

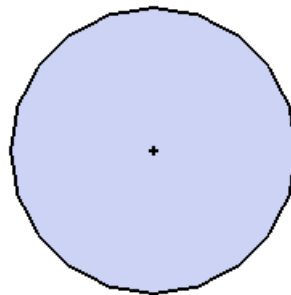
Teacher Note:

The answer is 20 edges. The number must be a multiple of 4 so that an endpoint will meet each quadrant point of the circle. The number must also be a multiple of 5 so that each pentagon corner will meet an endpoint.

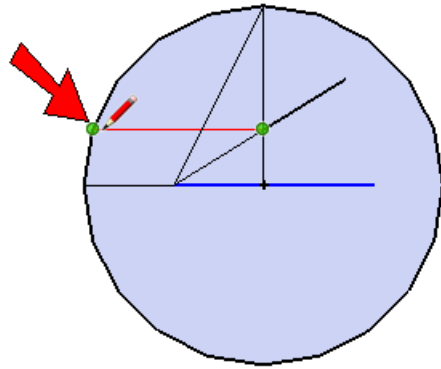


So the minimum number of edges is $4 * 5 = 20$. The 240-sided circle worked because 240 is a multiple of 20. Any multiple of 20 would work.

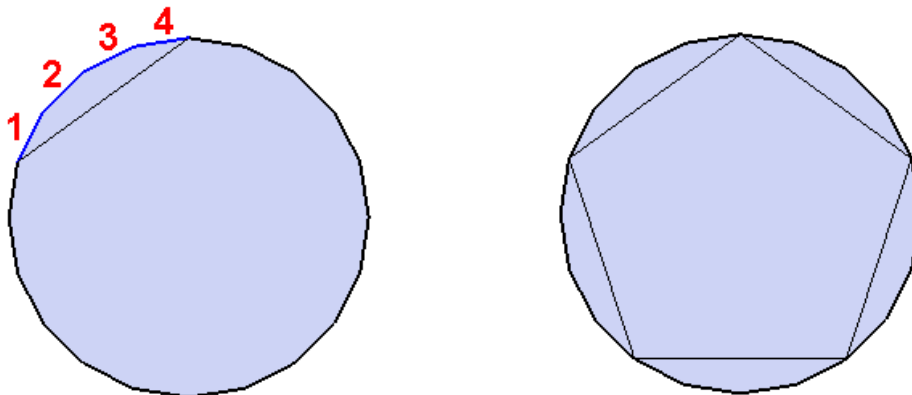
A 20-sided circle looks more like a polygon; you can see the individual endpoints.



The construction edges are easy to create since you can always see the endpoint each line is supposed to reach.



And when you complete the first pentagon edge, you'll see it's correct because its arc is comprised of 4 segments; one-fifth the total number of segments.



Teachers, Want More?

For more geometry projects using Google SketchUp, check out our GeomeTricks books. All books are available in print and as printable e-book. See <http://www.3dvinci.net/ccp0-catshow/GM.html>.

You can also sign up for our [SketchUp Project of the Month](http://www.3dvinci.net/ccp0-prodshow/POM.html) subscription. Each month you will receive three fun projects (one in math, two in 3D design) that can be used in K-12 classes. Details at <http://www.3dvinci.net/ccp0-prodshow/POM.html>. May has two projects that have geometric elements: one about the geometry of the baseball diamond, and one on the DNA helix.

