The Painted Cube, in Google SketchUp

Teacher Note: All text in red appears only in the teacher version, not in the student version.

This project is recommended for Grade 8 and up. In addition to helping students think and visualize in 3D, this is also a good exercise for finding patterns in tables of numbers.

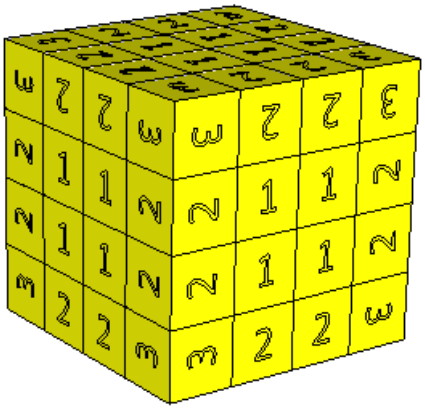
For this project, it helps to have some basic knowledge of Google SketchUp (though detailed instructions are provided). In particular, it's important to know how to zoom, orbit, and pan the view. If you need more information on how to get started, and a description of some basic tools, please read 3DVinci's Getting Started Guide (PDF).

PC users: go to http://www.3dvinci.net/SketchUp_Intro_PC.pdf.
Mac users: go to http://www.3dvinci.net/SketchUp_Intro_MAC.pdf.

For more educational materials based on SketchUp, please go to http://ww.3dvinci.net.

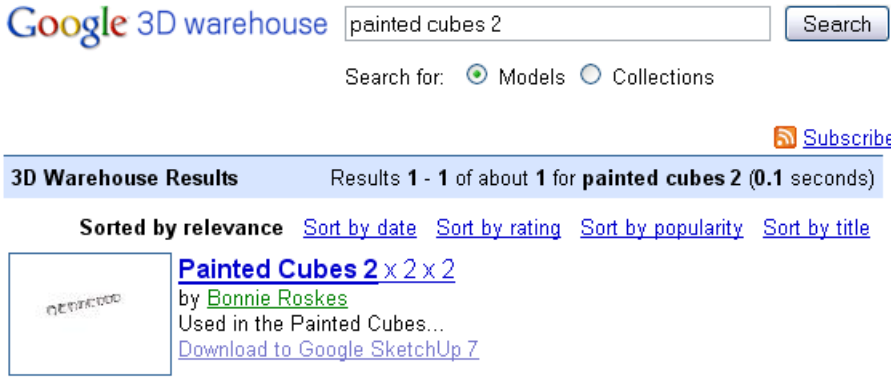
3DVINCI

You create a cube by joining and stacking smaller cubes. All of the outside faces are painted. What are the different ways the cubes can be painted, and how many of each cube will be needed?

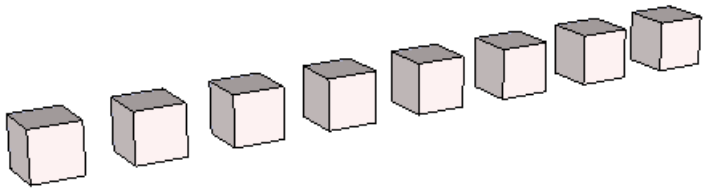


2 x 2 x 2 Box

1. To download the model used in this project, open the Google 3D Warehouse: http://sketchup.google.com/3dwarehouse. In the search field, enter “painted cubes 2.” For the model shown below, click the “Download to SketchUp” link.



Here is the model - eight identical cubes. The idea is to move and stack the cubes to form a larger cube (the box) which has 2 small cubes along each side.

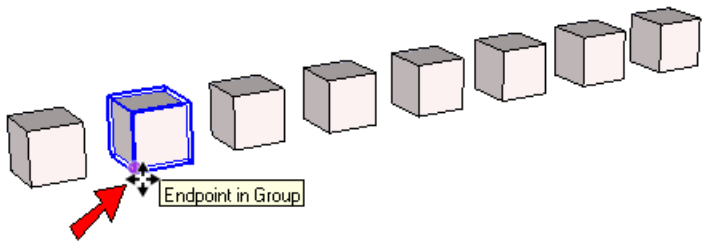


Each cube is a group, which means it acts as a single object (i.e. you don’t have to select all of its faces and edges). And because they are groups, the cubes can be moved together and then moved apart again, without them sticking to one another.

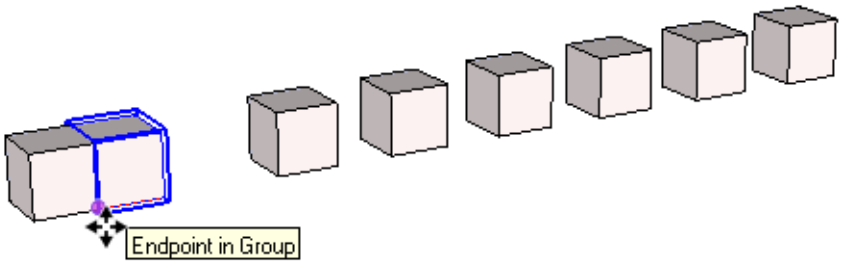
Once the box is formed, we’ll paint the outside faces of each small cube.

2. Activate the **Move** tool.

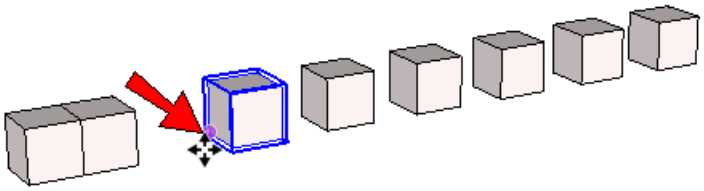
3. Click the lower left front corner of the second cube.



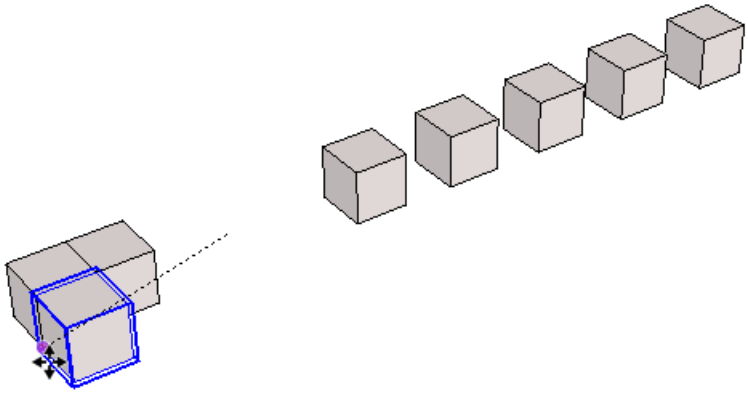
4. Then click the lower right front corner of the first cube. This joins the first two cubes together.



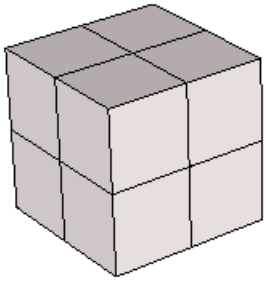
5. Keep the **Move** tool active, and click the back left corner of the third cube, as shown below.



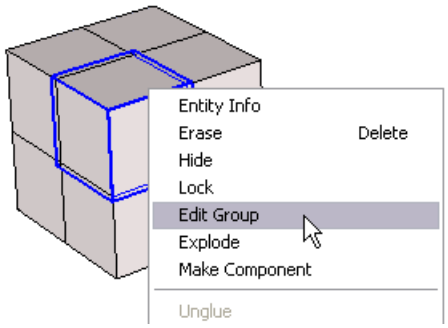
6. Move this cube in front of the first two cubes.



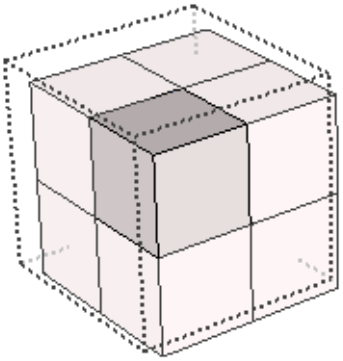
7. Move the remaining five cubes into place, to complete the box.



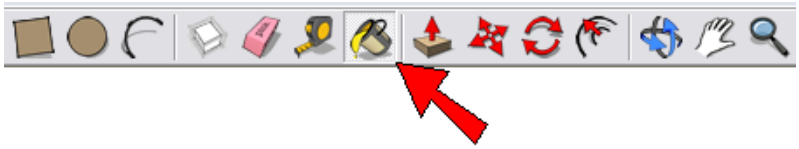
8. The next step is to paint the outside faces, but individual faces of a group can't be painted until the group is "opened" for editing. So right-click on one of the cubes and choose **Edit Group**.



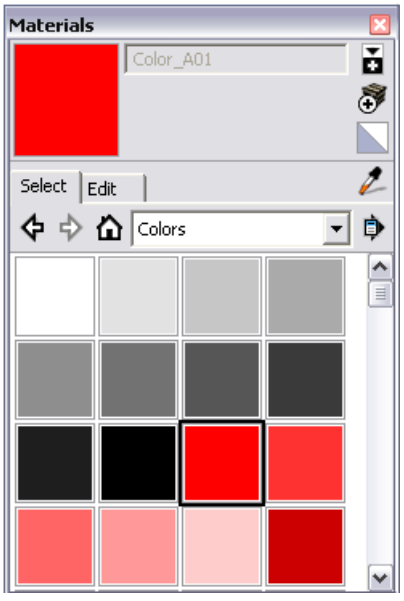
The opened cube can now be edited, and the other cubes are faded into the background.



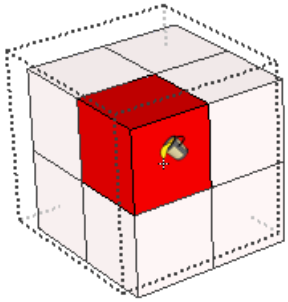
9. To find collections of colors and textures, click the **Paint Bucket**.



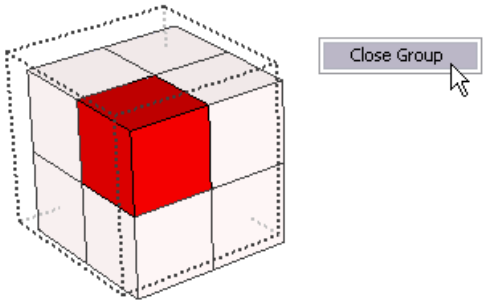
10. Find the collection you want and click the color square you want to use. (Mac users, you can either use one of the color picker options along the top of the **Colors** window, such as crayons, or click the brick icon to get a menu with texture and color collections.)



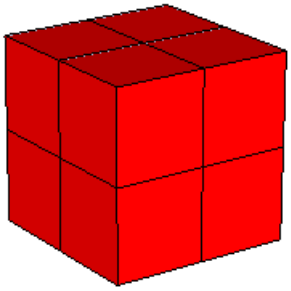
11. Click to paint each of the three outside faces of the opened cube.



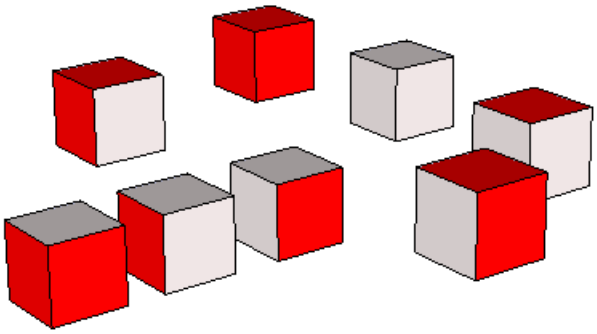
12. When finished, right-click in blank space and choose **Close Group**.



13. Open the remaining cubes and paint their outside faces. Each cube should have three painted faces.



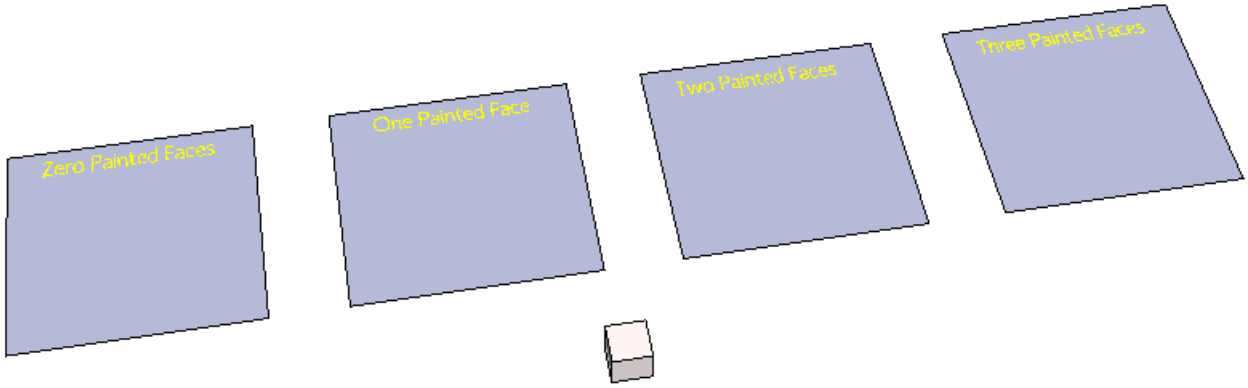
14. To make sure your cubes are painted correctly, activate **Move** again and separate them. Orbit around to make sure that each cube has three painted faces, and you'll see that each of the eight cubes has three different faces painted.



3 x 3 x 3 Box

The 2 x 2 x 2 box was easy, because all eight cubes were painted the same way (three painted faces). But when we make a larger box, we'll have four different painted cubes: some with three faces painted, some with two, some with one, and some with zero.

1. Go back to the 3D Warehouse and download the "Painted Cubes 3 x 3 x 3" model. This model contains just one cube, which we'll copy several times to complete the box. (As before, this cube is a group.) The model also contains four "mats" which we'll use when we separate the painted cubes by their number of painted faces.



   First we'll build the 3 x 3 x 3 box.

Question: How many total cubes will be needed?
Answer: 3 x 3 x 3 = 27.

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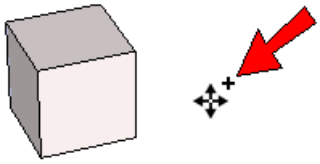
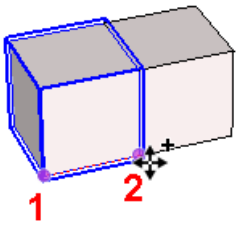
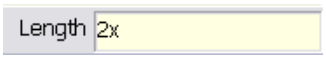
2. Activate **Move** again and press the Ctrl key, or Option on the Mac. (You don't have to keep this key pressed, just tap it once.) A "plus" sign now appears next to your cursor, which means we'll make copies.



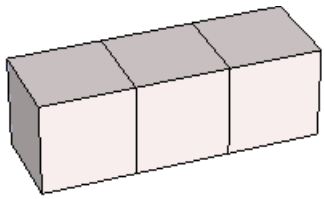
3. Click Point 1 and Point 2 shown below, which creates one copied cube right next to the original one.



4. We need one more copy, so type 2x, which appears in the **Length** field at the lower right corner of the SketchUp window. Then press Enter or Return.

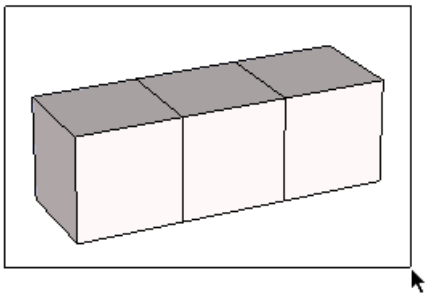


Now there are three cubes - the first row.

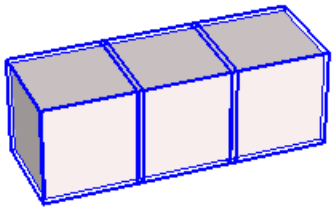


5. We now want to copy all three cubes to make two more rows. Because we'll be copying more than one object, the objects must all be selected before they can be copied. So activate the **Select** tool.

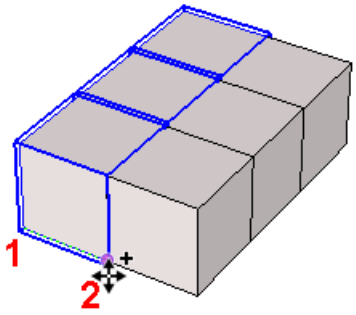
6. Click and drag a selection window that surrounds the three cubes, making sure the window doesn't touch anything else.



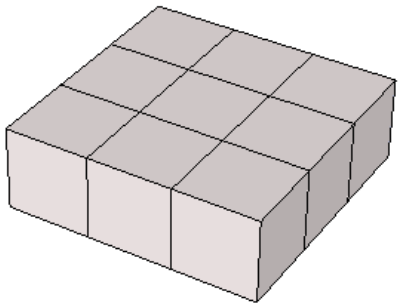
Now all three cubes are selected.



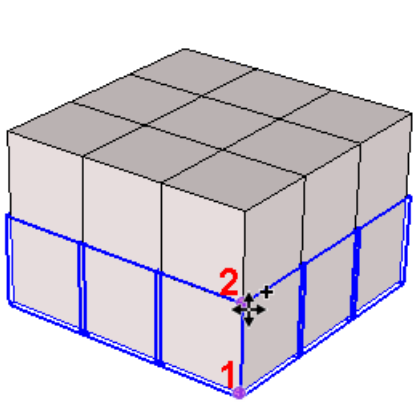
7. Activate **Move** again, press Ctrl / Option, then click Points 1 and 2 to create the second row.

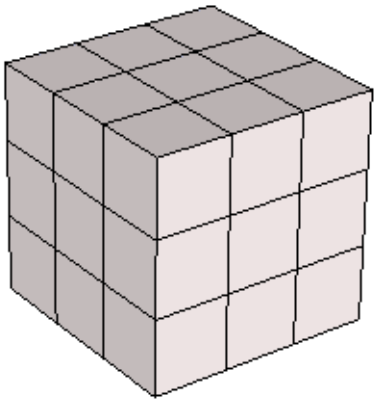


8. Type 2x and press Enter to add the third row. This completes the first layer of the box.

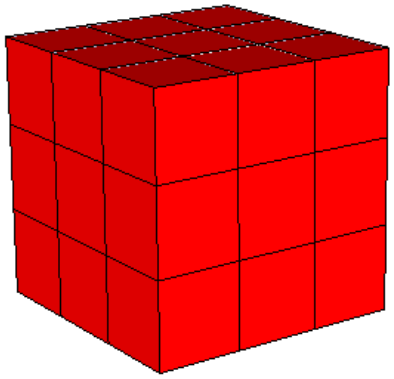


9. For the second level, select all of the cubes again and make the copy by clicking Points 1 and 2. Then enter 2x to complete the 3 x 3 x 3 box.





10. Like you did with the 2 x 2 x 2 box, open each cube group for editing and paint its outside faces.



Question: Without counting in your model, how many cubes have zero, one, two, and three faces painted?
Answer:
A box 8 corners, so 8 cubes should have three painted faces.
A box has 12 edges, and each edge contains 1 two-face cube. So 12 cubes should have two painted faces.
A box has 6 faces, and each face has 1 one-face cube in the middle. So 6 cubes should have one painted face.
There is 1 cube in the very middle of the box, with no faces on the outside. So 1 cube has zero painted faces.

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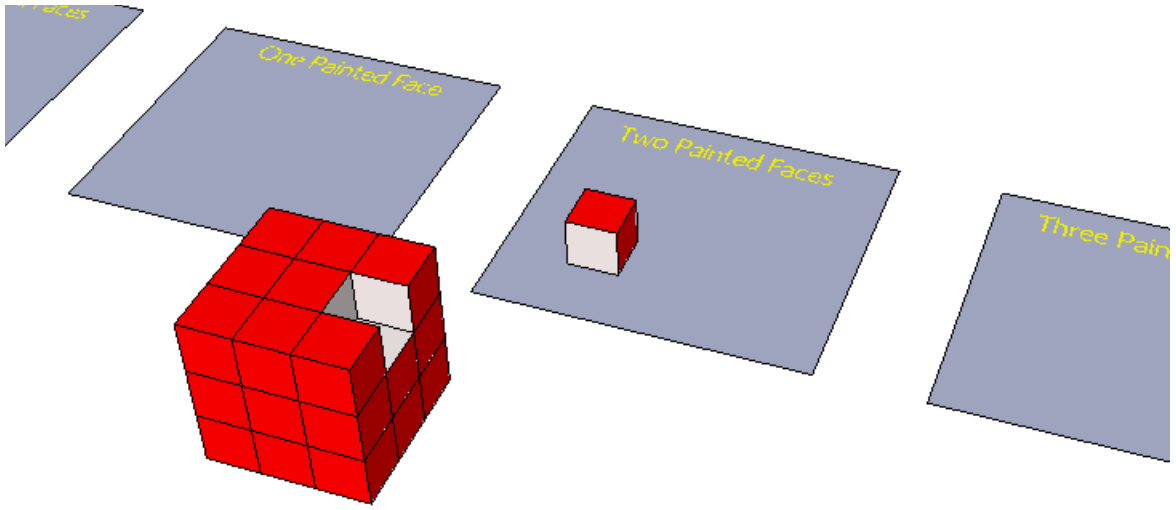
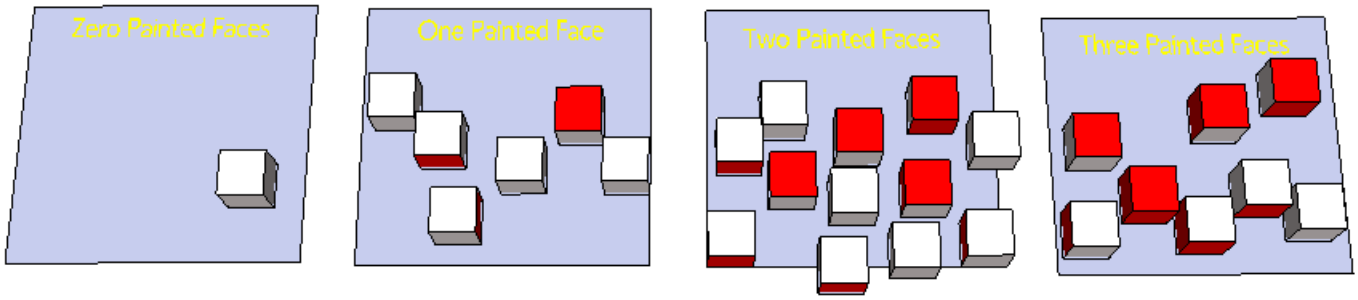
11. To see how many cubes of each painted type there are, use the **Move** tool to move each cube onto its correct mat, one at a time.



12. Here's how your mats should look after the cubes are separated: 1 zero-face, 6 one-face, 12 two-face, and 8 three-face. Add them up to get 27.



13. Create a chart that shows the numbers of each cube in each box. So far we've worked on the 2 x 2 x 2 and 3 x 3 x 3 boxes, and this is what we know:

# cubes per row (n)	zero painted faces	1 painted face	2 painted faces	3 painted faces	total cubes
2	0	0	0	8	8
3	1	6	12	8	27

4 x 4 x 4 Box

Now that we know the four different types of painted cubes that form a box, we'll create the 4 x 4 x 4 box with cubes that are already painted.

Question: How many total cubes will be used this time?
Answer: 4 x 4 x 4 = 64

Question: Can you guess how many of each painted cube will be used?
Answer:
8 corners (as always), so 8 three-face cubes.
12 edges, each with 2 two-face cubes. So 24 of these.
6 faces, each with 4 one-face cube. So 24 of these.
In the middle, there are 2 x 2 x 2 = 8 zero-face cubes

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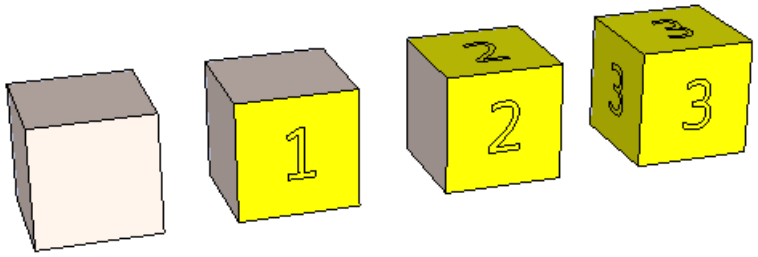
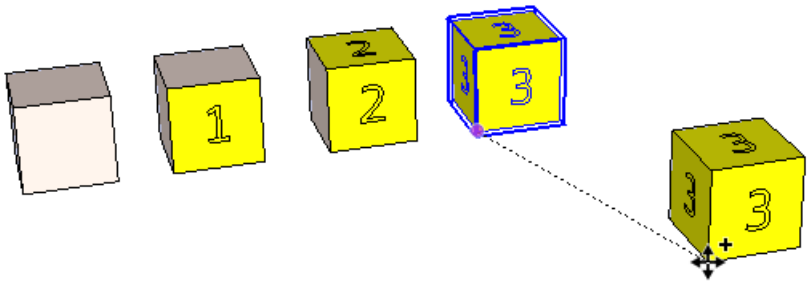
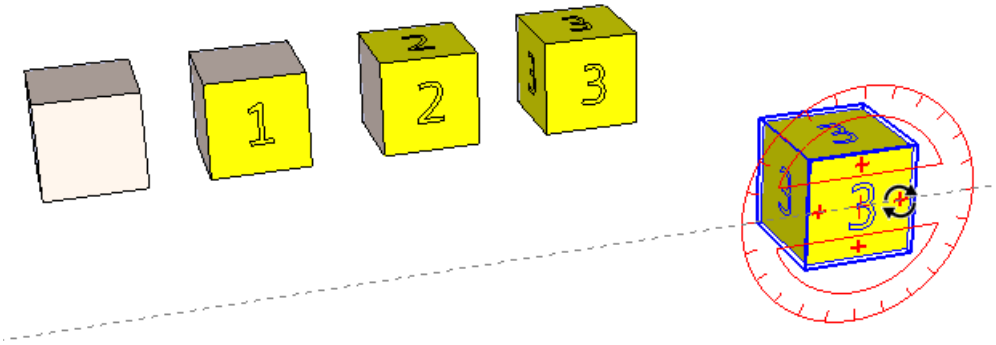
1. From the 3D Warehouse download the "Painted Cubes 4 x 4 x 4" model. This model contains the four cubes you'll need, with yellow faces that will go on the outside of the box. This time each cube is a component instead of a group. This makes it easy later to tally the total number of each cube used.



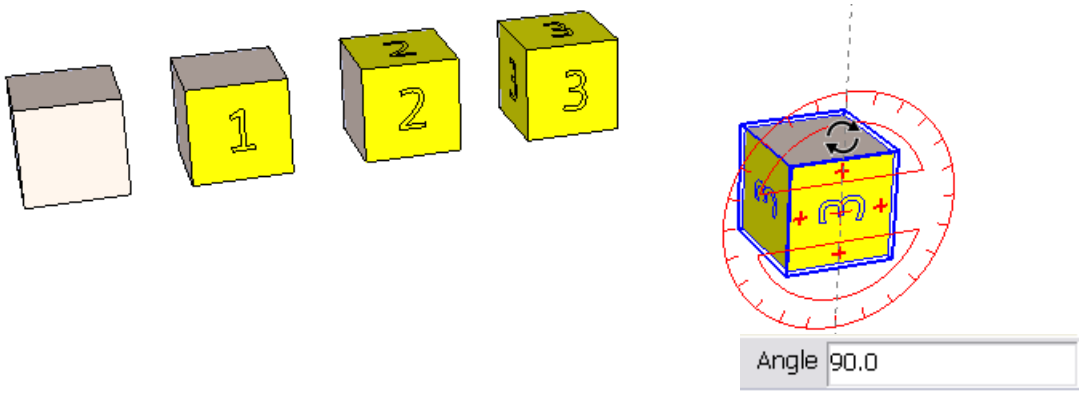
2. We'll keep our "pool" of four cubes off to the side, and use copies to build the box. We'll start with the bottom of the box, with a corner cube. So make a copy of the "3" cube.



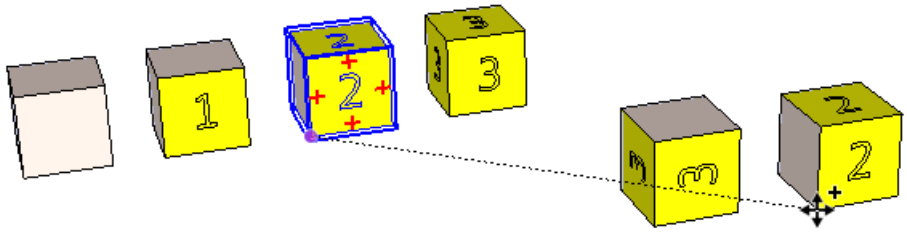
3. For this cube to be on the bottom, it needs to be spun around so that a yellow face will be on the bottom. While **Move** is the active tool, you can both move and rotate a component. Move the cursor to the front face of this cube, and click on one of the red "plus" signs. These are rotation handles.



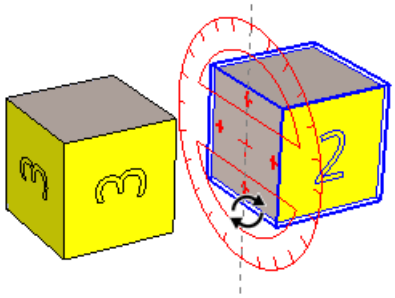
4. Move the mouse to spin the cube 90 degrees counter-clockwise. While the cube is spinning, watch the **Angle** field at the lower right corner of the SketchUp window. Your angle may be different than the 90 degrees shown below; it depends on which rotation handle you clicked. Be sure to click when the value is 0, 90, 180, or 270 (it may also be negative).



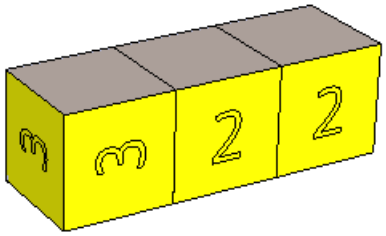
5. Now bring over a copy of the "2" cube.



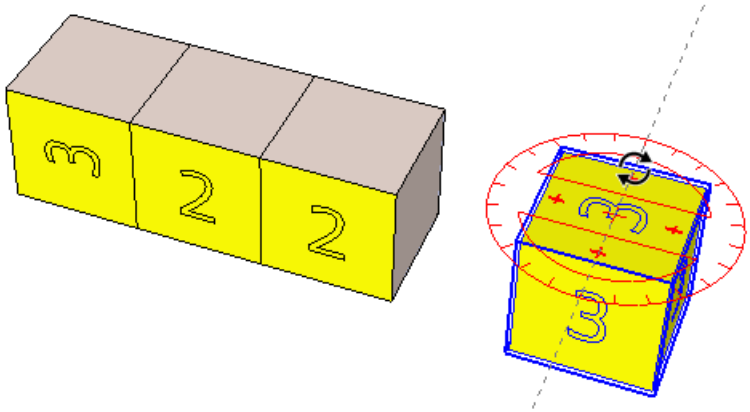
6. Click a rotation handle on the side face shown below, and rotate this cube 90 degrees clockwise.



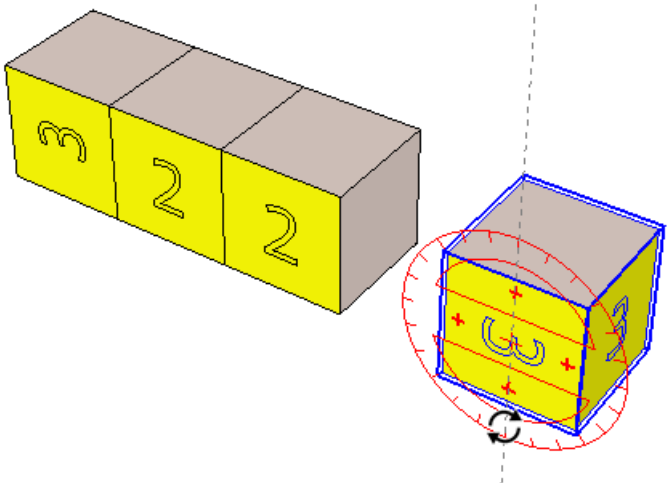
7. Move the "2" into place, and place another "2" next to it, rotated the correct way.



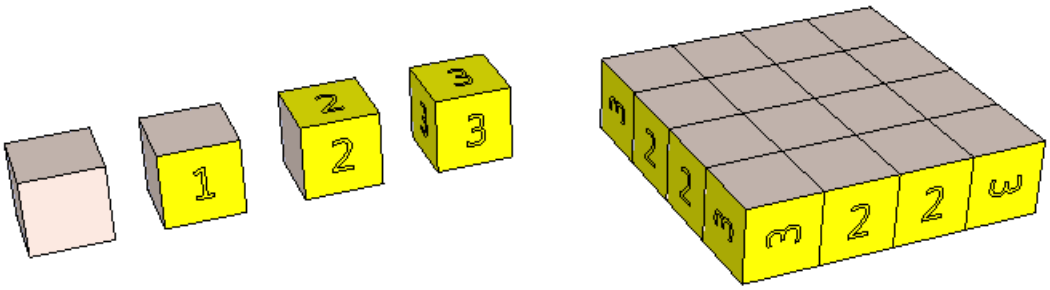
8. To complete the row, bring over another “3” for the corner. This one will need to be rotated twice. First, rotate the top face 90 degrees counter-clockwise.



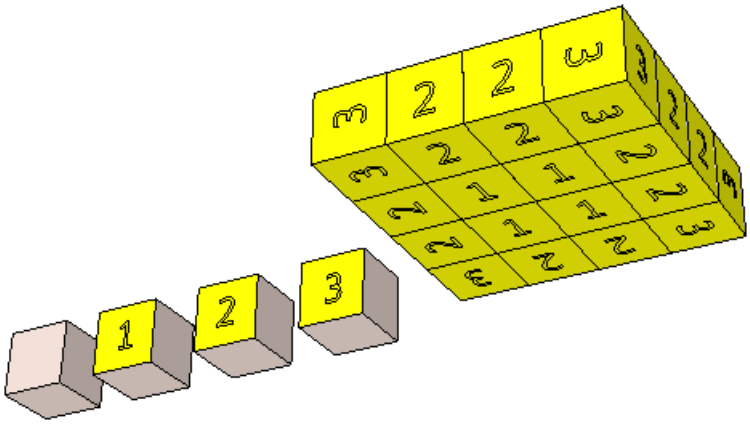
9. The rotate the front face 90 degrees clockwise.



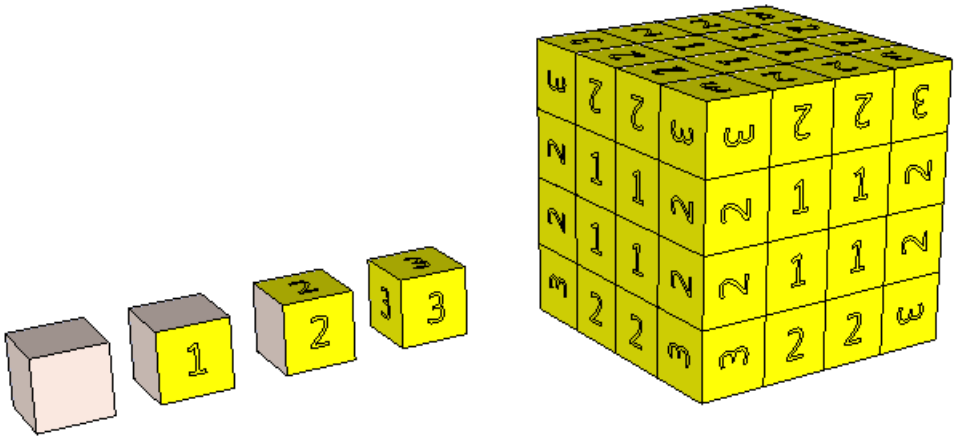
10. Move the “3” cube into place, and continue copying, rotating, and moving cubes to complete the bottom layer. Don’t forget to use “1” cubes for the four cubes in the middle.



11. Orbit to check the bottom and sides of this layer, to make sure that only yellow faces are showing.



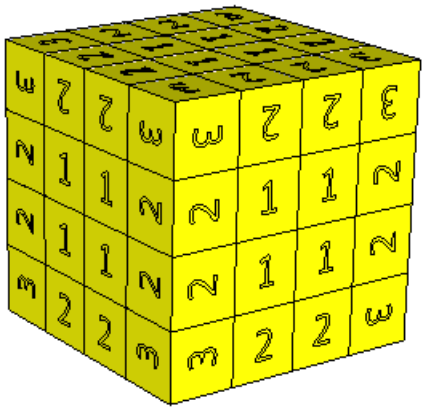
12. Finish the other three layers of the box.



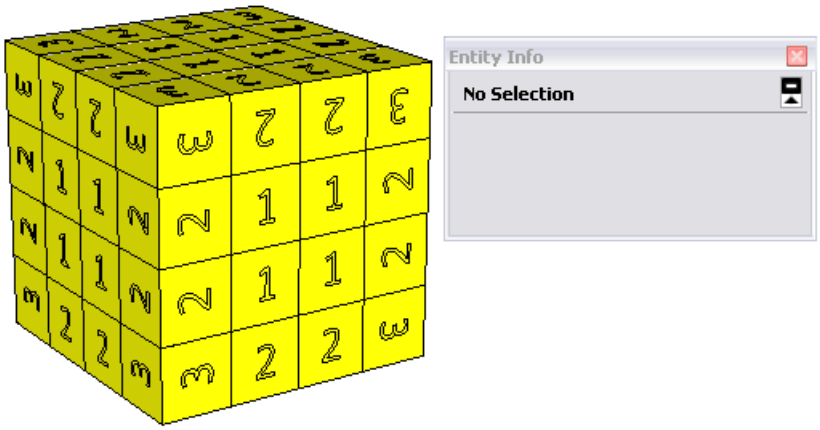
*Tip: If you know how to use the **Rotate** tool for copying, you can save a lot of time.*

13. The last task is to tally the number of cubes used. But the original four cubes are still there, and we don't want them included in the counting. So click the **Eraser** tool.

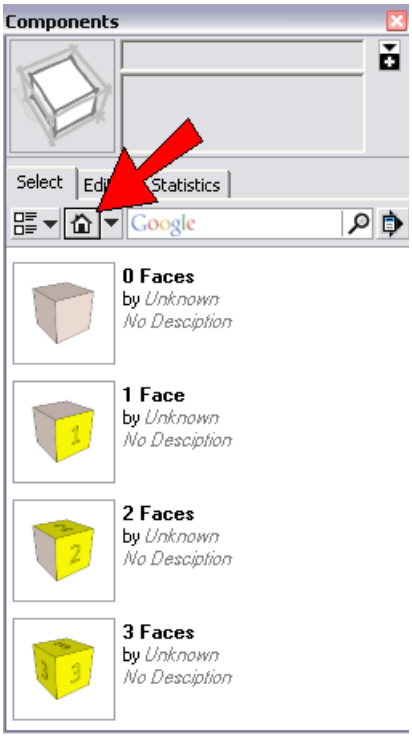
14. Click each of the four original cubes to erase them. Now only the cubes used in the box are left.



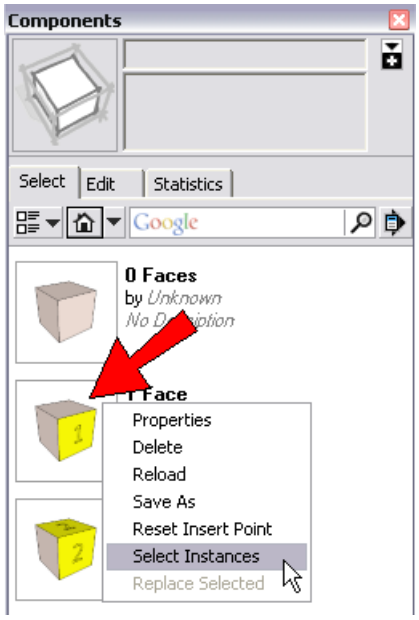
15. From the main menu, choose **Window / Entity Info**. This opens the **Entity Info** window, which should be empty since no objects are selected.



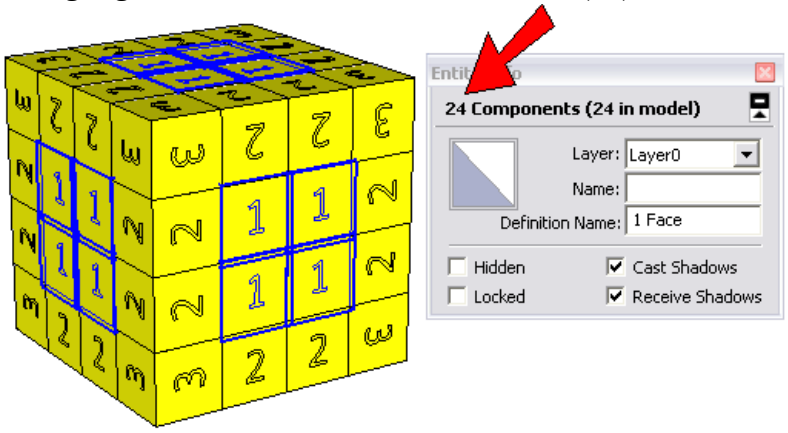
16. Then choose **Window / Components**, which opens the **Components** window. Click the house icon, which shows a list of the four components used in the model.



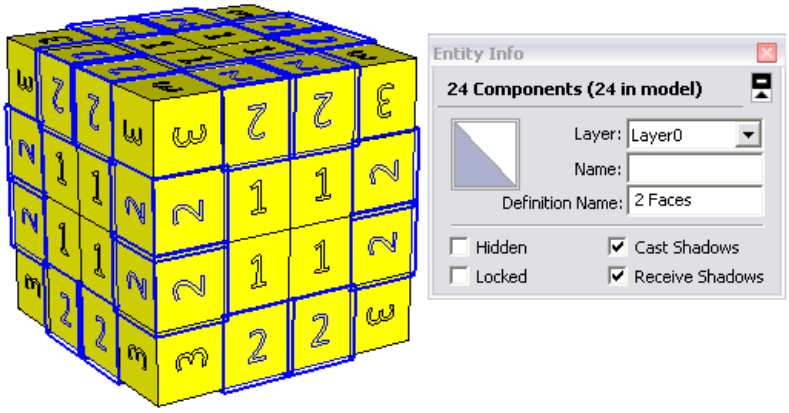
17. Right-click on the “1 Face” component, and chose **Select Instances**.



All of the “1” cubes are highlighted, and the total number of them (24) is listed in the **Entity Info** window.



18. In the **Components** window, right-click on the “2 Faces” component and choose **Select Instances**. Again, there are 24.

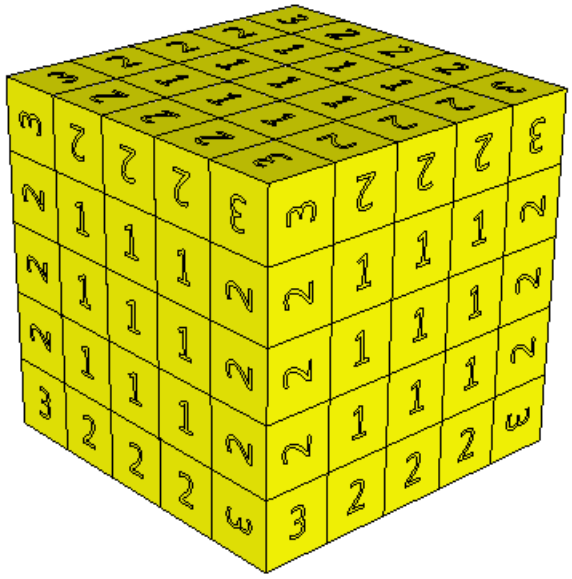


19. Do the same to tally the “3 Face” cubes, and the “0 Face” cubes (even though you can’t see them selected because they’re on the inside). Add these numbers to your chart.

# cubes per row (n)	zero painted faces	1 painted face	2 painted faces	3 painted faces	total cubes
2	0	0	0	8	8
3	1	6	12	8	27
4	8	24	24	8	64

5 x 5 x 5 Box

Start over with the model you used for the 4 x 4 x 4 box, and use the same steps to build a 5 x 5 x 5 box. How many cubes of each type will be needed?



Answer:
Total cubes: 125
Three-face cubes: 8
Two-face cubes: 3 x 12 = 36
One-face cubes: 9 x 6 = 54
Zero-face cubes: 3 x 3 x 3 = 27

Continue filling numbers in your cube tally chart. What pattern can you find in the numbers of cubes with respect to n? Can you predict what how many cubes will comprise a 6 x 6 x 6 box?

After completing the 5 x 5 x 5 box, here is what your tally chart should contain:

# cubes per row (n)	zero painted faces	1 painted face	2 painted faces	3 painted faces	total cubes
2	0	0	0	8	8
3	1	6	12	8	27
4	8	24	24	8	64
5	27	54	36	8	125

Zero-face cubes: $(n-2)^3$

One-face cubes: $6(n-2)^2$

Two-face cubes: $12(n-2)$

Three-face cubes: 8

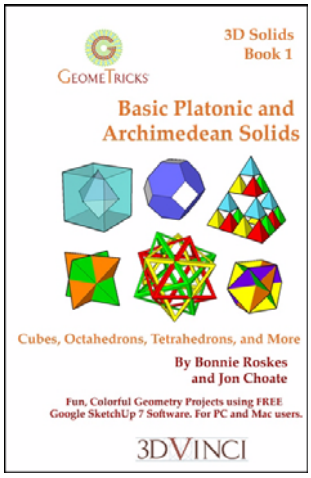
These values are the product of a binomial expansion: $n^3 = ((n-2) + 2)^3$

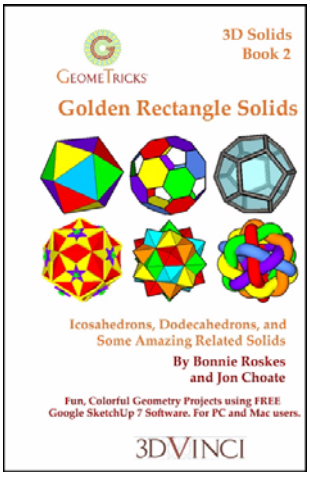
And the expansion of the binomial is $n^3 = (n-2)^3 + 6(n-2)^2 + 12(n-2) + 8$

So, for a 6 x 6 x 6 box, you will have 64 zero-face cubes, 96 one-face cubes, 48 two-face cubes, and 8 three-face cubes, for a total of 6x6x6 = 216 cubes.

Want More?

If you like 3D geometry projects in Google SketchUp, you'll love 3DVinci's GeomeTricks series! The two books shown below contain 3D projects on Platonic and Archimedean solids:





All books are available in print and as printable PDF. For details on GeomeTricks, including 2D geometry project books, go to http://www.3dvinci.net/ccp0-catshow/GM.html.

Also, be sure to sign up for our SketchUp Project of the Month subscription! Each month you will receive THREE FUN PROJECTS (one in math, two in 3D design) that can be used in K-12 classes. Details at http://www.3dvinci.net/ccp0-prodshow/POM.html. The project for September 2009 is on Escher triangles: