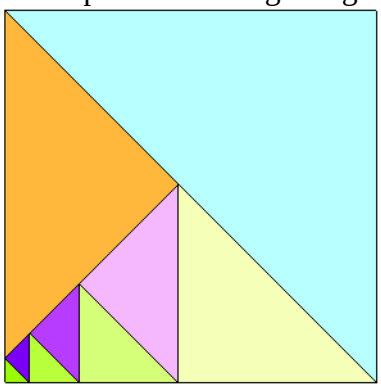
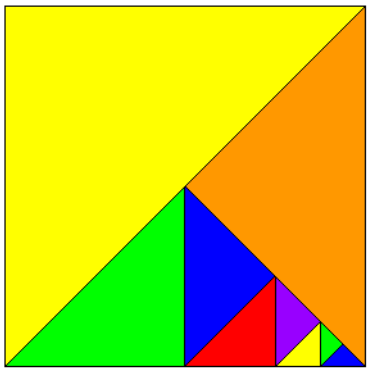
Right Isosceles Triangles Form a Square, in Google SketchUp

A right isosceles triangle has two equal sides, and interior angles of 45-45-90. There are many ways to combine a set of right isosceles triangles into a square, and this project shows a few possibilities. We'll also look at the patterns and relationships between edge lengths and areas of adjacent triangles.

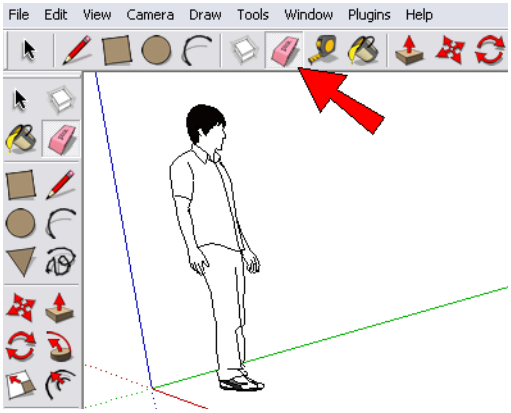




Assemble the Triangles

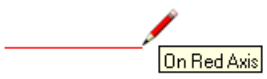
We'll start with one right isosceles triangle and use it as the basis to make the rest of the triangles.

1. Open Google SketchUp. If your file contains a person standing on the ground near the origin, click the **Eraser** tool and erase him.



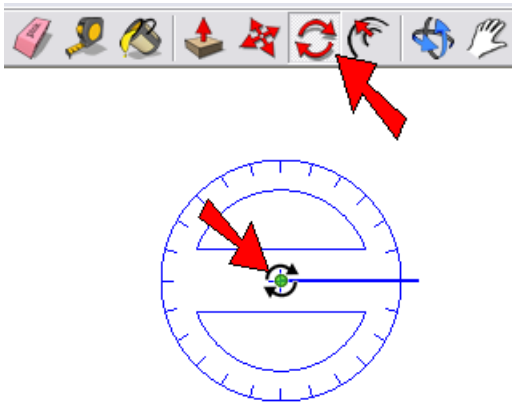
2. From the main menu, choose **Camera / Standard Views / Top**. Now you're looking down on the "ground," and the word **Top** appears in the top left corner of the SketchUp window.

3. We'll start the first isosceles triangle with a single line. Click the **Line** tool (hotkey: L), and draw a horizontal line of any length. Press Esc when the line is finished.

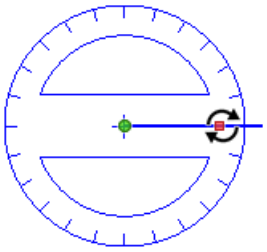


4. This line will be copied at a 90-degree angle. First, click the **Select** tool (hotkey: Space Bar), and click the line you just drew.

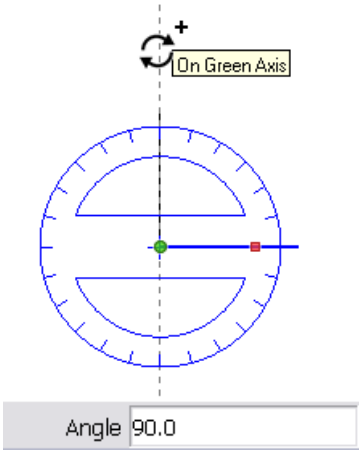
5. With the line selected, activate the **Rotate** tool. Place the protractor at the left endpoint of the line.



6. To establish the angle baseline, click anywhere along the line.

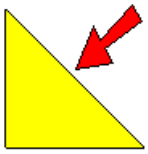


7. Because we're copying this line, press (tap, don't hold) the Ctrl key (PC users) or the Option key (Mac users). Then move the mouse so that the copied line is straight up, in the green direction. Look in the **Angle** field at the lower right corner of the SketchUp window; the angle should be 90 degrees. Click to complete the copy.

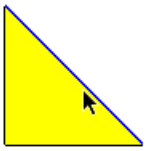


*If you ever find that you can't get the angle you want by just moving your mouse, you can always type the angle (don't click in the **Angle** field; just type) and press Enter.*

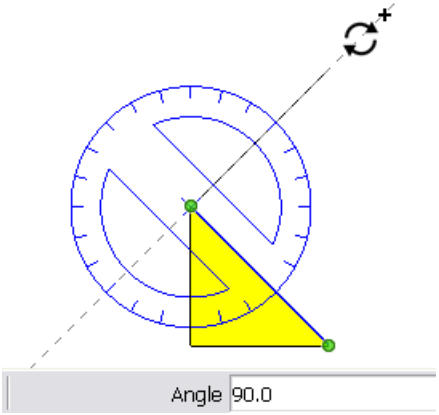
8. Use the **Line** tool to complete the third edge of the triangle, which fills in the triangle with a face. (If you want to paint your triangles, click the **Paint Bucket** tool and open one of the collections of colors or materials.)



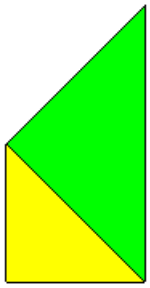
9. The third edge you just added (the triangle's hypotenuse) will be used to create the next triangle. Activate **Select** and click this edge.



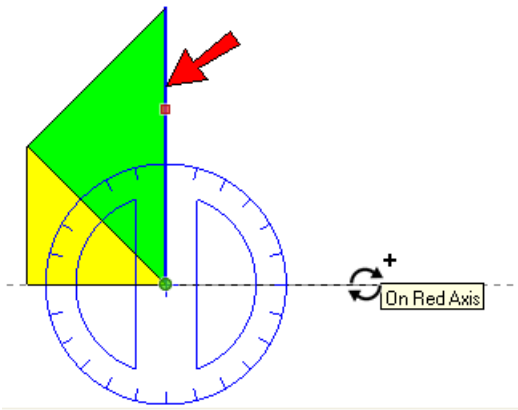
10. Use the **Rotate** tool to make a 90-degree copy of this edge, as shown below. (Don't forget to use the Ctrl/Option key to make the copy.)

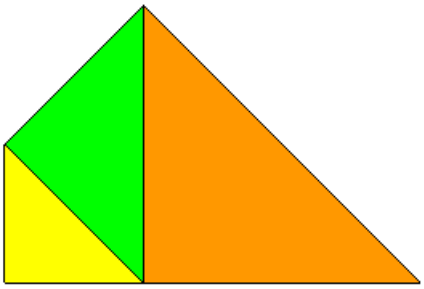


11. Add the hypotenuse for the second triangle (which should be perfectly vertical), and now there are two right isosceles triangles.

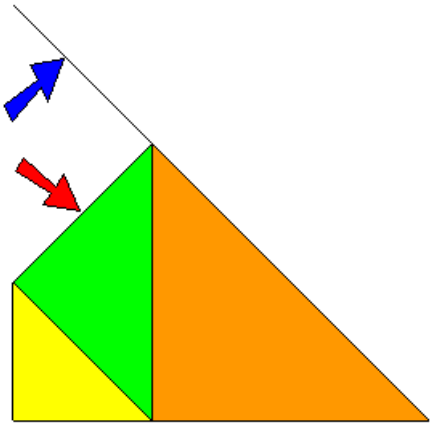


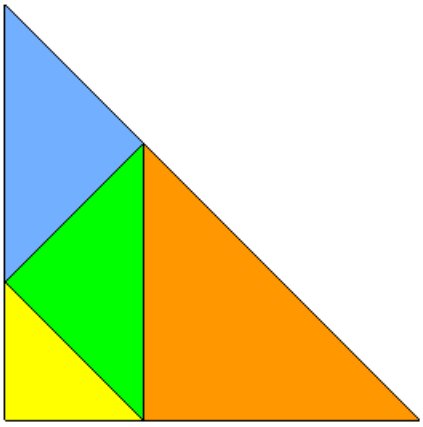
12. Once again, the new hypotenuse will be used to create the next triangle. Select this edge and make a 90-degree copy (the copied edge should be perfectly horizontal). Then add the new hypotenuse to complete the third triangle.



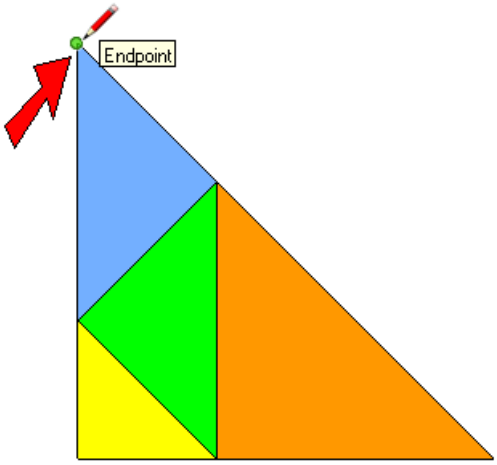


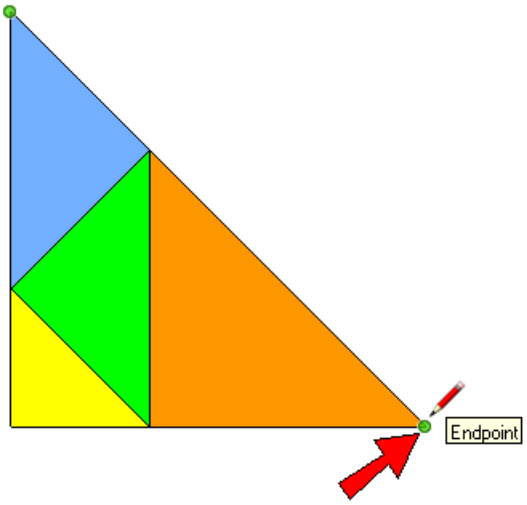
13. For the next triangle, select the edge indicated below by the red arrow, and copy it 90-degrees to make the edge indicated by the blue arrow. Add a hypotenuse to this triangle.



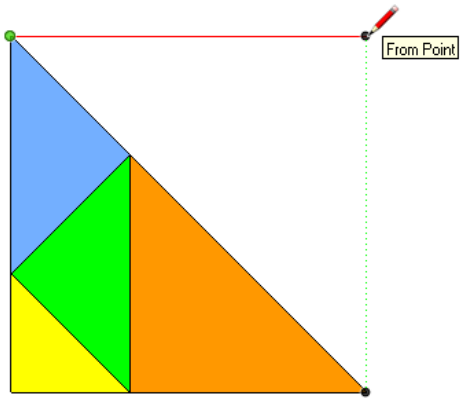


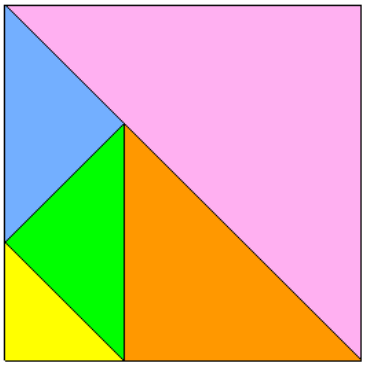
14. Half of the square is already filled in, and there's just one more triangle needed to complete the square. This time we already have the hypotenuse of the new triangle, but we need the other two equal-length sides. Start a line at the point shown below on the left, and hover (don't click) over the point show below on the right.





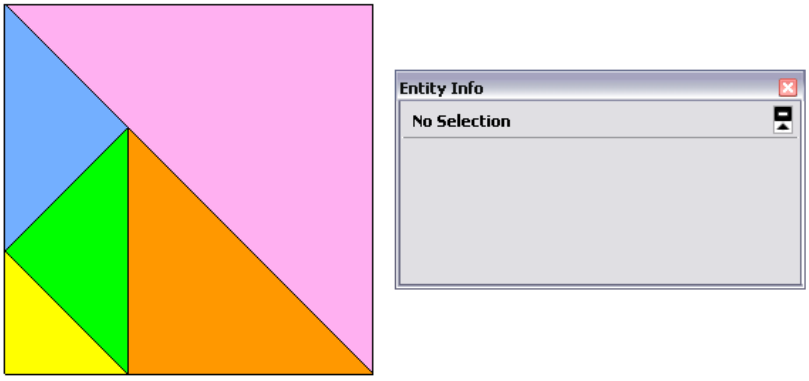
15. Move the mouse straight up until you see red and green "helper lines," then click to finish the horizontal edge. Draw one more edge straight down to complete the triangle.



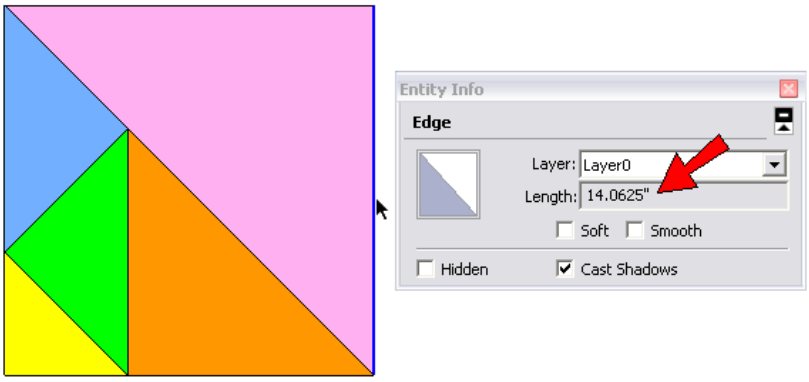


Is It a Square?

1. The only way to know for sure whether we've created a square is to measure its width and height. So from the main menu, choose **Window / Entity Info**, which opens the **Entity Info** window. This is where we can check lengths and areas.



2. Activate **Select** and click the vertical edge of the largest triangle. The length of this edge appears in the **Entity Info** window.



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*The measurement appears in whatever units are set for your file. You can change the units by choosing **Window / Model Info** and opening the **Units** page. **Decimal** units work best for taking measurements, especially when you're working in Imperial units (feet-and-inches).*

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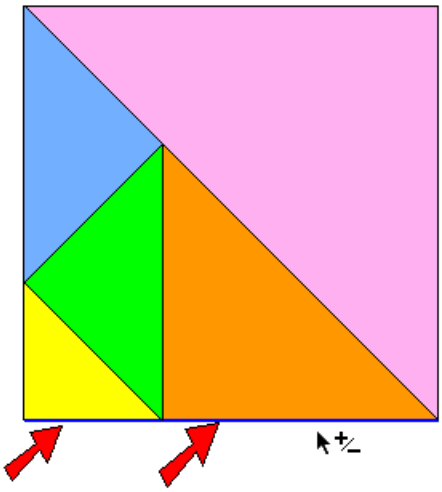
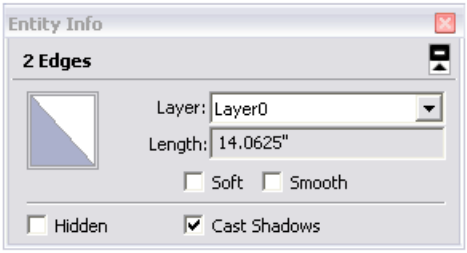
3. The horizontal length of the square can be measured along either the top edge or bottom edge. If you're going to use the bottom edge, keep in mind that there are two parts to this edge, and both parts need to be selected. To select more than one edge at a time, press and hold the Shift key. (If you select a face by mistake, just click it again to unselect it.) The measurement you get should be the same as the one you got before.

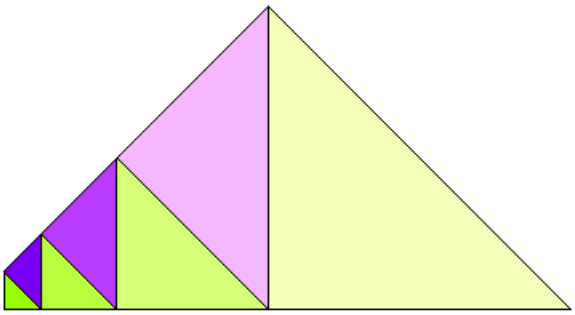




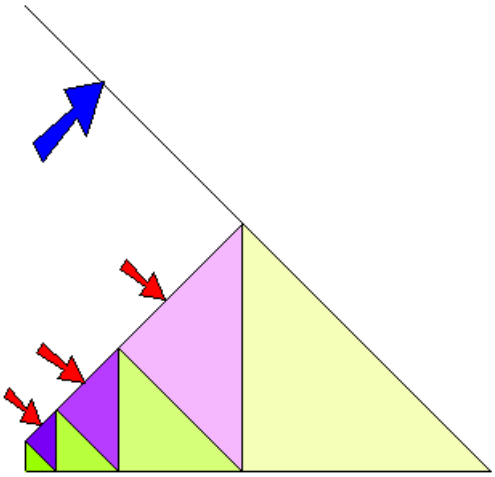
A Square with More Triangles

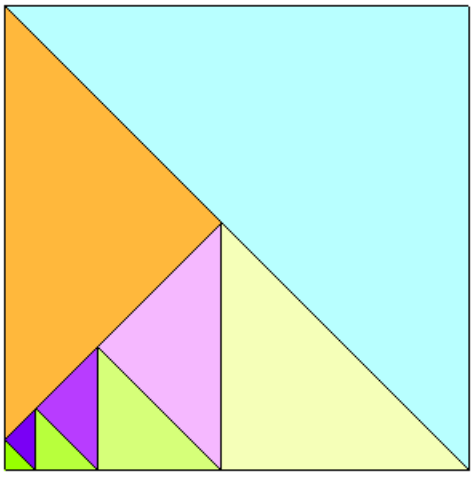
The square in the previous section was made of five triangles, but you can use different numbers of triangles, as long as you follow the correct pattern.

Start with the small green triangle shown below on the left side. Add a slightly larger purple, then a slightly larger green, and keep going until you end with a green. In other words, make an odd number of triangles.



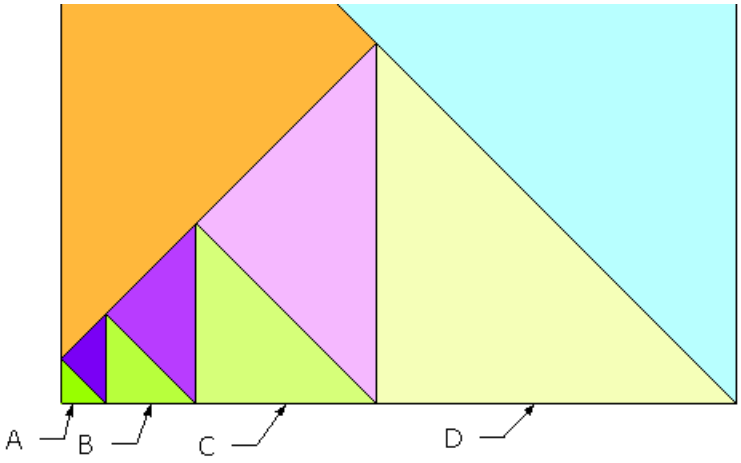
For the second-to-last triangle, select all of the outer edges of the purple triangles (indicated below by the red arrows). Make a 90-degree copy of the entire line of edges, resulting in the single edge indicated below by the blue arrow. Finish that triangle, then add the largest triangle to complete the square.



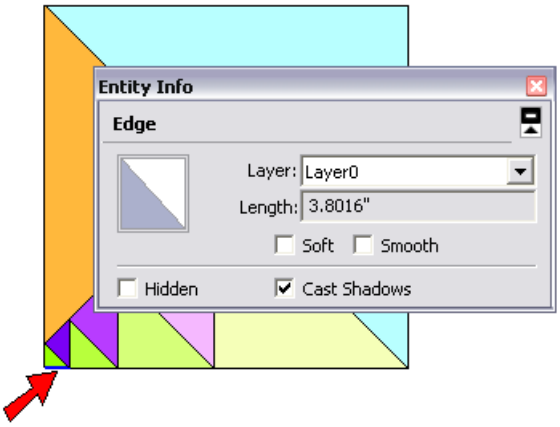


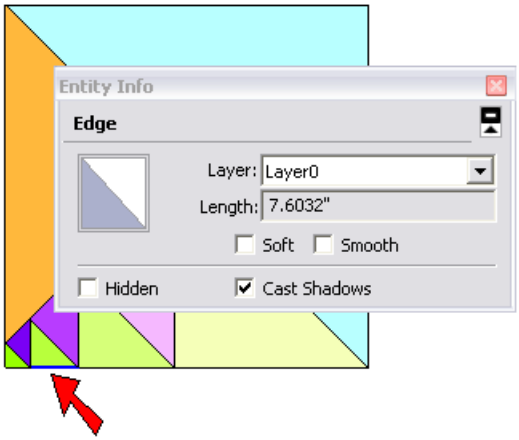
Patterns of Length and Area

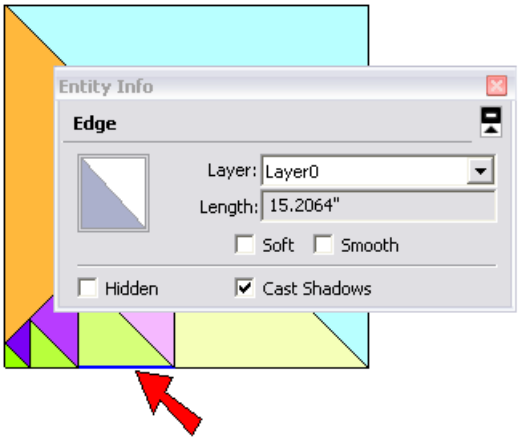
The relationship between edges follows a pattern. What is the relation between the lengths of Edges A and B? Between Edges B and C?

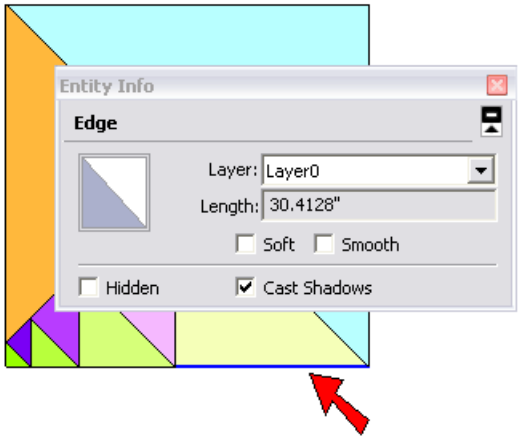


Using the **Entity Info** window and selecting these edges one by one, we can see that each edge is twice as long as the one directly to the left. Why?

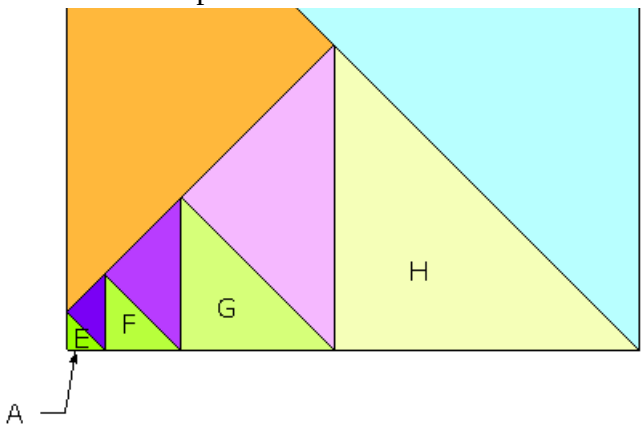




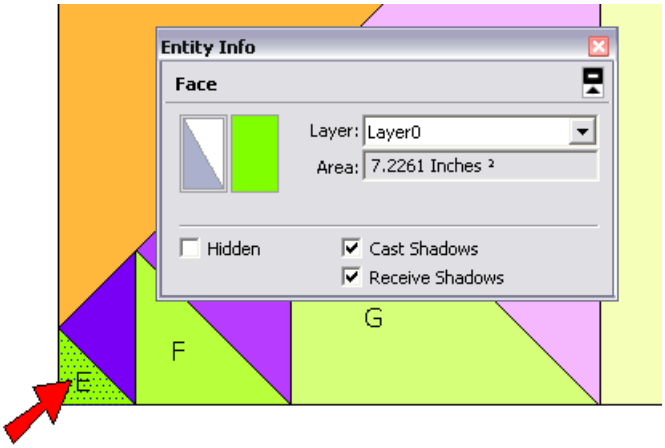


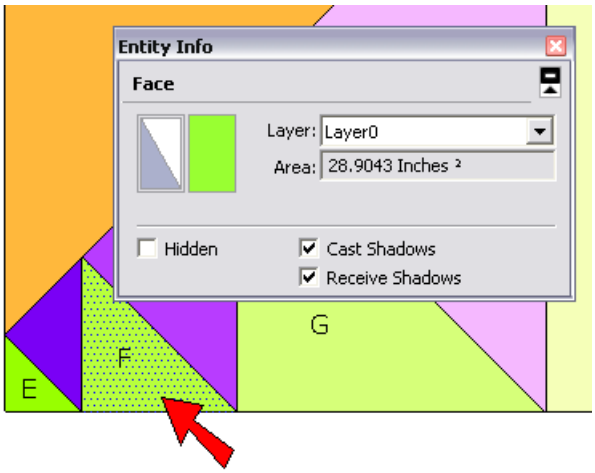


Now we'll look at area: what is the relationship between the areas of Face E and Face F? Between Faces F and G?

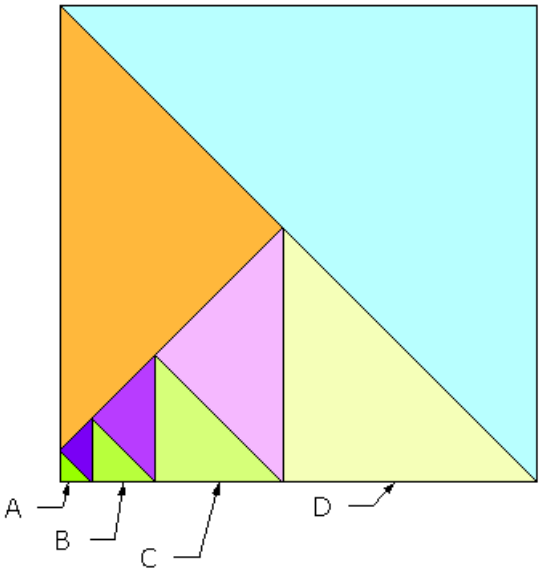


To measure area, select a face (not an edge) and **Entity Info** window tells you its area. Each face is four times as large as the one directly the left. Why?





One last question: If Edge A shown below is 1 unit long, what is the total area of the square?



Try This

There are several ways you can fit right isosceles triangles into a square. Can you make this one? Can you think of others?