Conic Sections in Google SketchUp

Teacher Note: All text in red appears only in the teacher version, not in the student version.

SketchUp's **Section Plane** tool is a treasure. It was designed for enabling interior views of houses (SketchUp is originally an architectural application, after all), but it is also a great tool for helping students visualize 3D geometry.

For a (slightly different) video version of this project, play the "Conic Sections" video at http://mathforum.org/sketchup/videos.html.

Younger students can create the 3D and 2D objects, since only a few simple tools are required. Older students who are learning about equations of curves can also take advantage of SketchUp's **Text** tool, to see whether their curves follow the correct paths. (The equations part is described only in the Teacher version of this project, not the Student version. It requires more SketchUp proficiency, so the steps are described loosely - a full explanation would double the size of this document!)

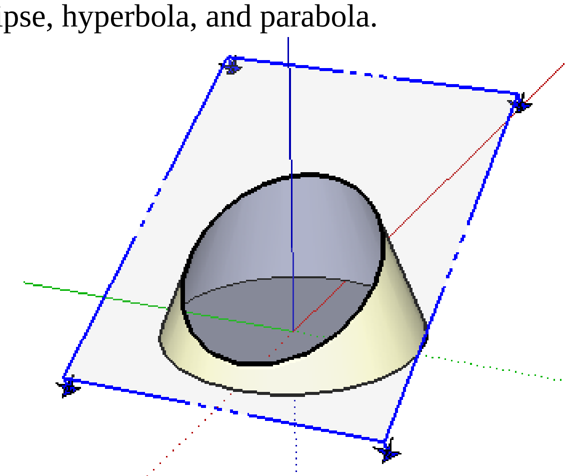
For this project, it helps to have some basic knowledge of Google SketchUp (though detailed instructions are provided). In particular, it's important to know how to zoom, orbit, and pan the view. If you need more information on how to get started, and a description of some basic tools, please read 3DVinci's Getting Started Guide (PDF).

PC users: go to http://www.3dvinci.net/SketchUp_Intro_PC.pdf.
Mac users: go to http://www.3dvinci.net/SketchUp_Intro_MAC.pdf.

For more educational materials based on SketchUp, please go to http://ww.3dvinci.net.

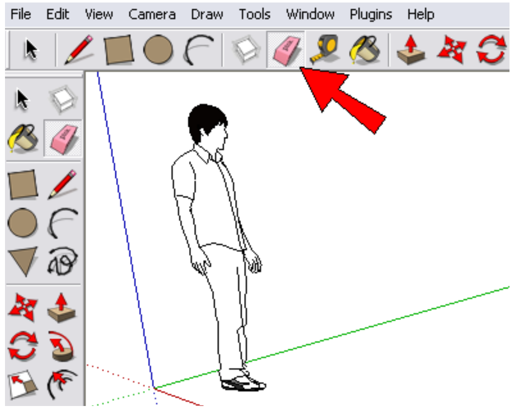
3DVINCI

In this project, we'll start with a basic 3D shape: a cone. From that one object, you can use section planes to create four different curves: a circle, ellipse, hyperbola, and parabola.



Step 1: Create the Cone

1. Open Google SketchUp. If your file contains a person standing on the ground near the origin, click the **Eraser** tool and erase him.



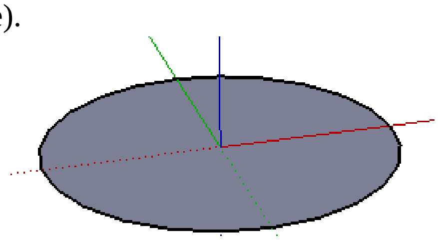
2. We need to see the axes for this project (the red, green, and blue lines), so if yours aren't displayed, choose **View / Axes** from the main menu.

3. Many of the tools for this project aren't available in the default toolbar that runs across the top of the SketchUp window. For the more complete toolbar (which includes most, but not all SketchUp tools), choose **View / Toolbars / Large Tool Set** (PC) or **Views / Tool Palettes / Large Tool Set** (Mac).

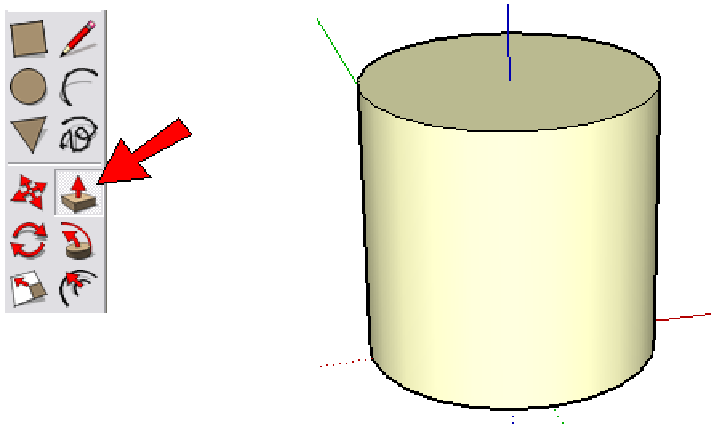
4. A cone is derived from a cylinder, which is derived from a circle. So click the **Circle** tool.

Teacher Note: if you're going to try some equation math later (described at the end of this document), it's important to understand that SketchUp's circles aren't true circles; they're actually segmented polygons. Before you click the center point, the field at the lower right corner of the SketchUp window reads **Sides**, which is set by default to 24. You'll get a closer approximation to a circle if you give it more sides. So if you want to increase the number of sides, type the number you want (don't click in the **Sides** field, just type), and press Enter. Caution: If you use a number that's too high (like 1,000) the rest of the project might move a little slowly!

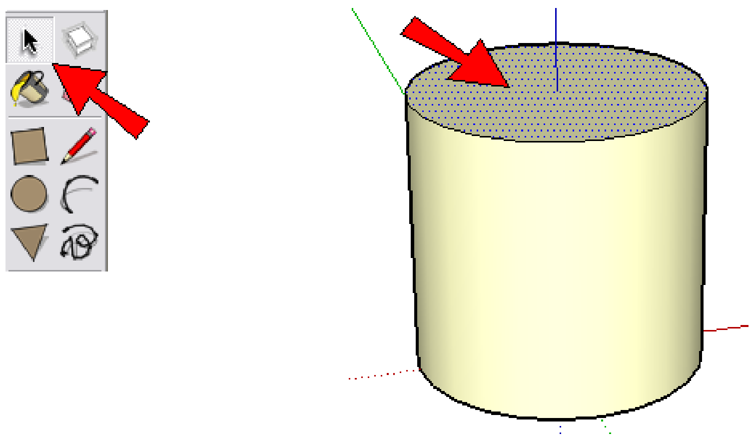
5. Place the center of the circle at the origin (where the three axes meet), and create a circle of any size sitting on the ground (the red-green plane).



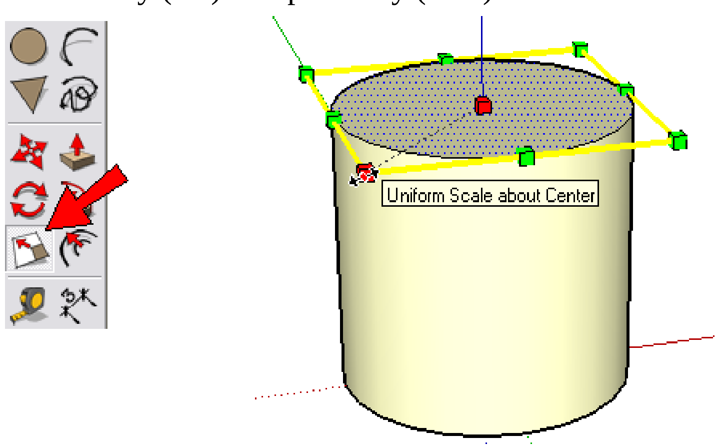
6. To make the cylinder, activate the **Push/Pull** tool. Then click the circle (the face, not an edge), move the mouse up, and click again when the cylinder has the height you want.



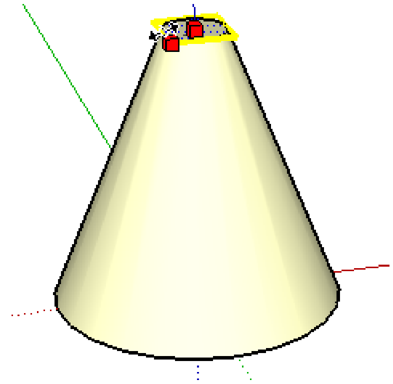
7. Now we can change the cylinder into a cone, by shrinking the top face. First, activate the **Select** tool, then click the top face of the cylinder to select it.



8. With the top face still selected, activate the **Scale** tool. A set of green boxes, called "drag handles," appears around the face. To shrink this face about its center, place the cursor on one of the corner handles (don't click yet), and press and hold the Ctrl key (PC) or Option key (Mac).

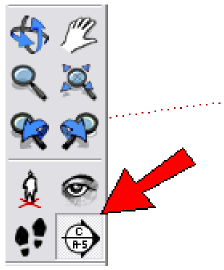


9. Click and drag this handle to shrink the face. Don't move your cursor too far, or you'll turn the face inside out!

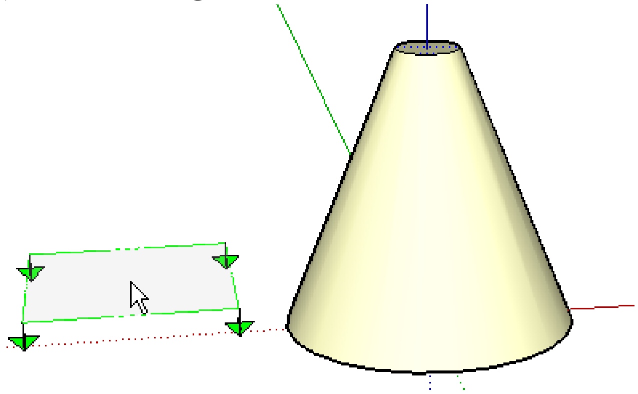


Step 2: Circle and Ellipse

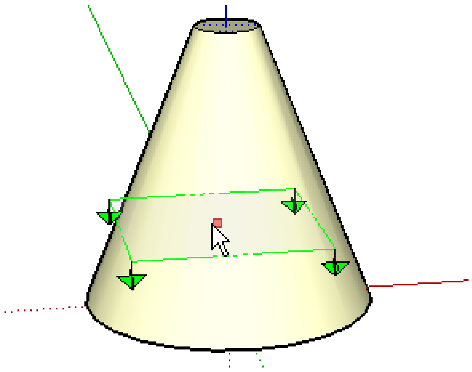
1. We'll now slice through the cone. Activate the **Section Plane** tool.



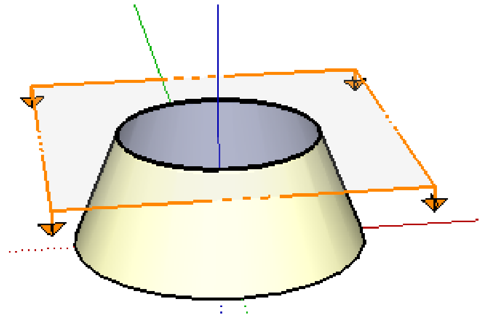
2. Keep the cursor in blank space, so that the section plane preview (which is green with four downward-pointing arrows) is flat on the ground.



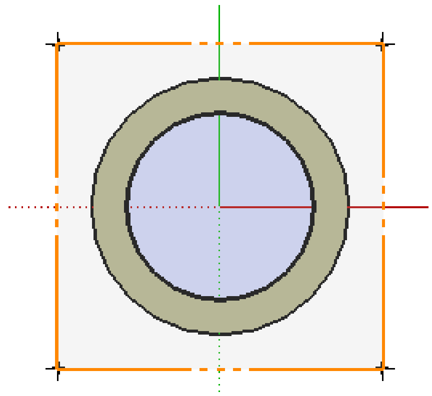
3. Press and hold the Shift key, which keeps the section plane flat. Then move your cursor somewhere on the cone.



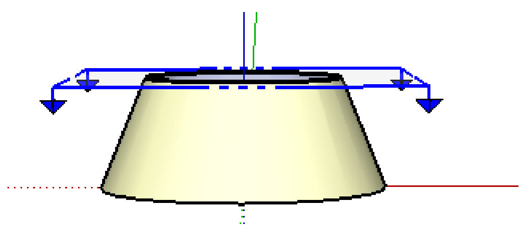
4. With Shift still pressed, click to create the section. Now the top part of the cone is sliced off, and you can see inside.



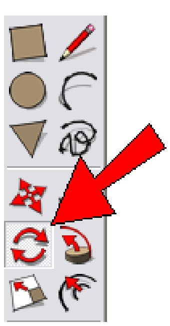
5. Orbit so that you are looking straight down into the cone. Where the cone meets the section plane you should see a circle (pretty obvious, right?).

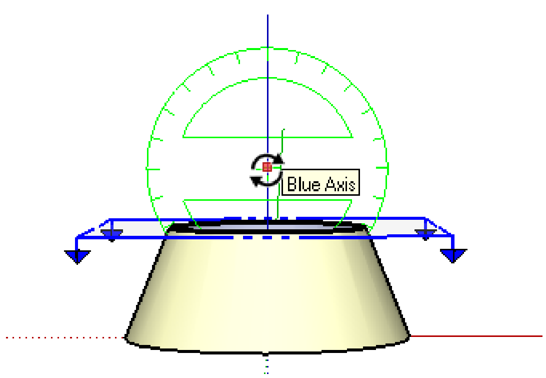


6. To change the circle into an ellipse, the section plane needs to be tilted. First do two things: activate **Select** and click the section plane (it should turn blue when selected), and orbit so that you're facing the side of the cone, looking straight along the green axis. The red axis should go off to the right and left.

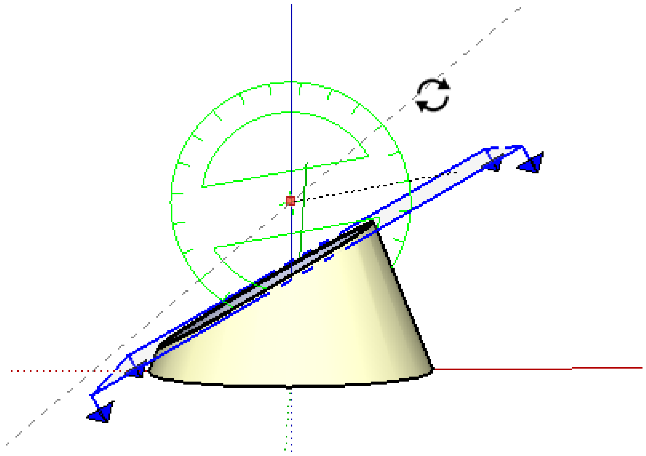


7. Activate **Rotate**. The protractor appears, and should be green. (If it's not green, move your cursor up a little until the protractor turns green, then press and hold the Shift key to keep it green.) Click to place the protractor on the blue axis, just above the section plane. This point defines the center of rotation.

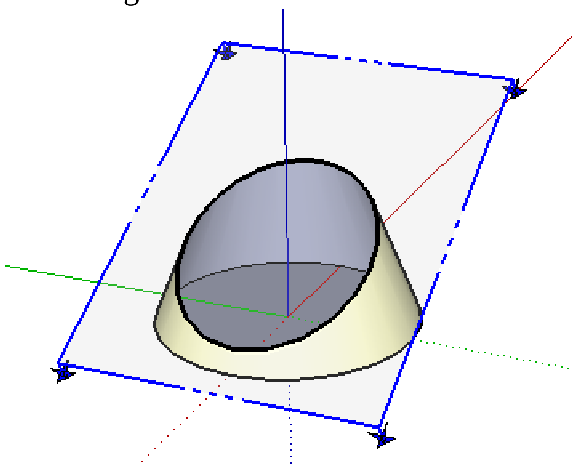




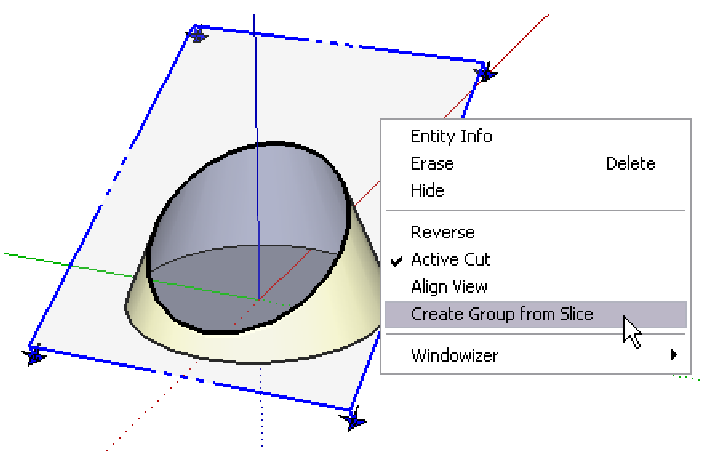
8. Then click two points that define the rotation angle. Make sure you don't go past the bottom of the cone.



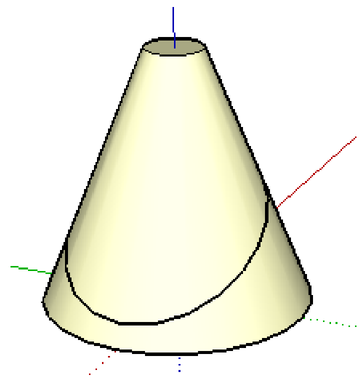
Your model should now look something like this. The curve within this section plane is an ellipse.



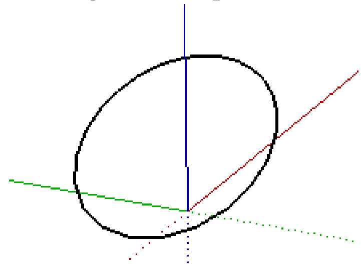
9. To save this ellipse, right-click anywhere on the section plane (not within the cone itself), and choose **Create Group from Slice**.



10. The section plane is no longer needed, so click the **Eraser** and erase the section plane (click on one of its edges, not in the middle). Now you have the cone and the curve.

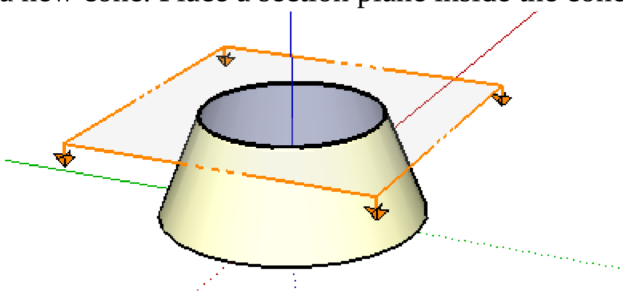


11. Erase the cone, which you can do by clicking on its top and bottom edges. Now just the ellipse is left.

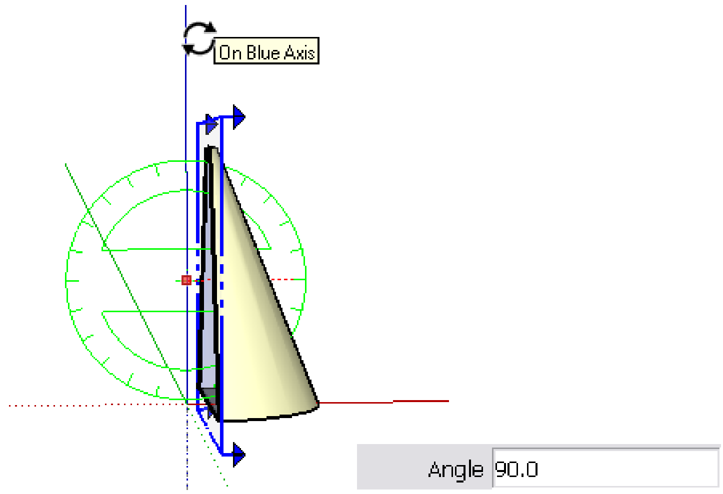


Step 3: Hyperbola

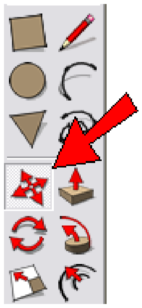
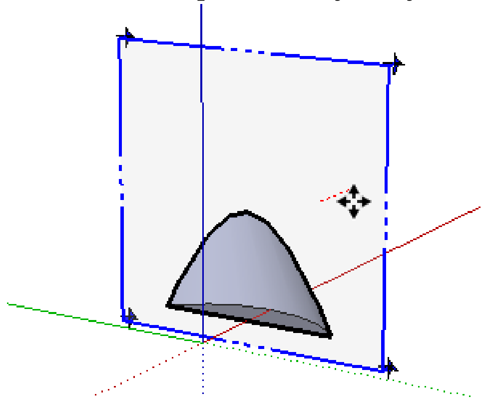
1. Start a new file, and create a new cone. Place a section plane inside the cone, like you did before.



2. Select the section plane, activate **Rotate**, and this time make the section plane vertical. This means the rotation angle should be 90 degrees, which you can check in the **Angle** field.



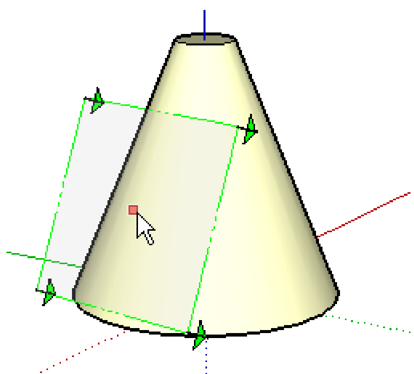
3. This creates one hyperbola, but you can move the section plane back and forth within the cone, to get differently hyperbolas. To do this, make sure the section plane is still selected, and activate **Move**. Click the section plane and move your cursor back and forth - the plane always stays oriented the same way.

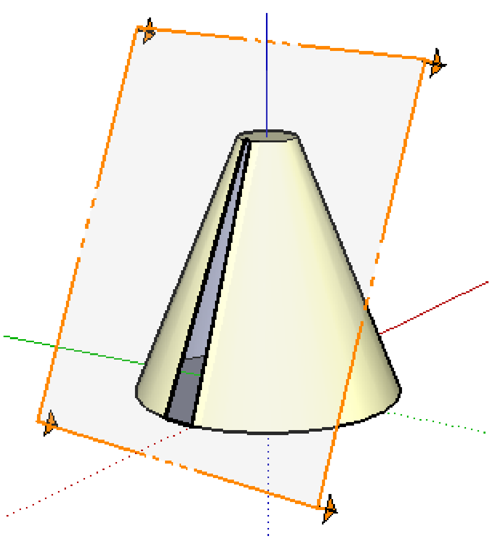
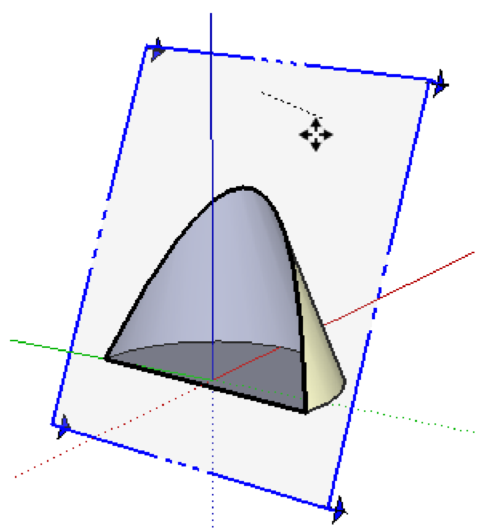
Step 4: Parabola

A parabola is created when you slice through a cone in a direction *parallel to the slope of the cone*. So we need to section this cone a different way.

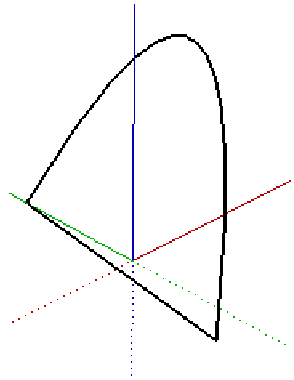
1. Once again, start a new file and create a new cone. Activate **Section Plane**, but this time don't press the Shift key to keep it flat. Instead, move your cursor around the cone; the section plane aligns to whatever face the cursor is on.



2. Click any face to section it. Then select the section plane and use the **Move** tool to push the plane farther into the cone.

3. Use the **Create Group from Slice** option, then erase both the section plane and cone. Here's your parabola, plus a horizontal line at the bottom:

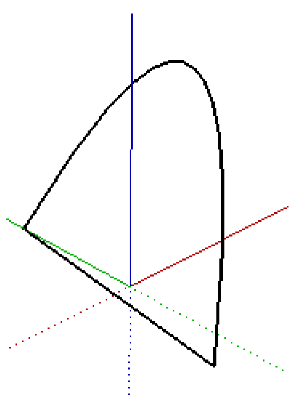


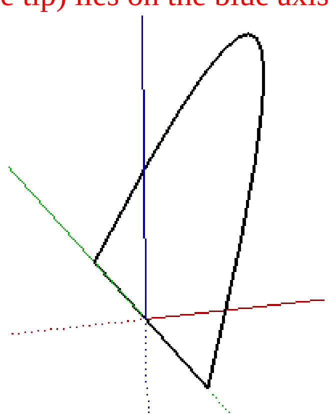
Conic Section Equations

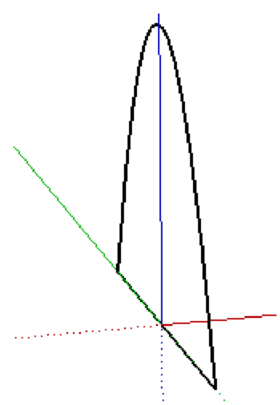
For students who are studying equations of conic sections, SketchUp enables you do to some interesting number-crunching. As stated at the beginning of this project, starting out with a highly segmented circle will yield the most numerically accurate results.

In addition to familiarity with the conic equations, this requires some proficiency with SketchUp. I haven't listed each step in detail, but if you've used SketchUp's basic drawing and editing tools, you should be able to pull this off!

First, let's look at a parabola. The first picture below shows what your parabola might look like after it's created by parallel sectioning a cone, as described above. The second picture shows the parabola rotated so that it starts and ends along the green axis, and centered at the origin. The third picture shows the parabola rotated again to be vertical, so that the vertex of the parabola (the tip) lies on the blue axis.







If the coordinates of the vertex are (h, k), then this is the curve equation:

$y = a (x - h)2 + k$

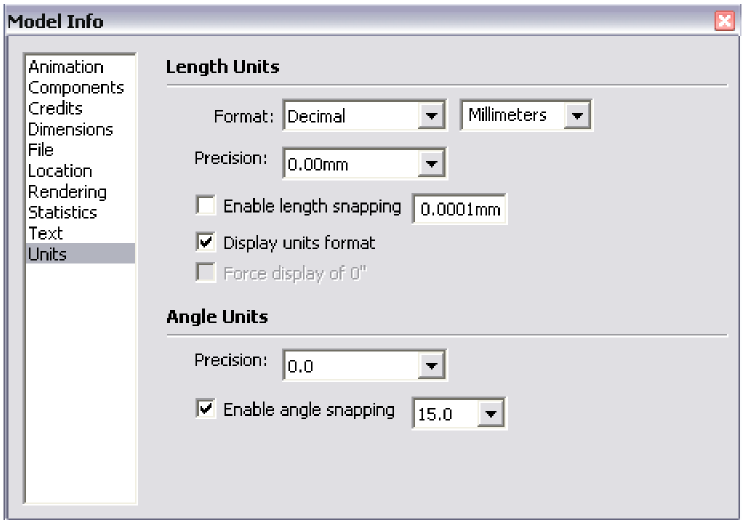
Because the parabola is centered on the origin, $h = 0$. So the equation becomes

$y = ax2 + k$

or

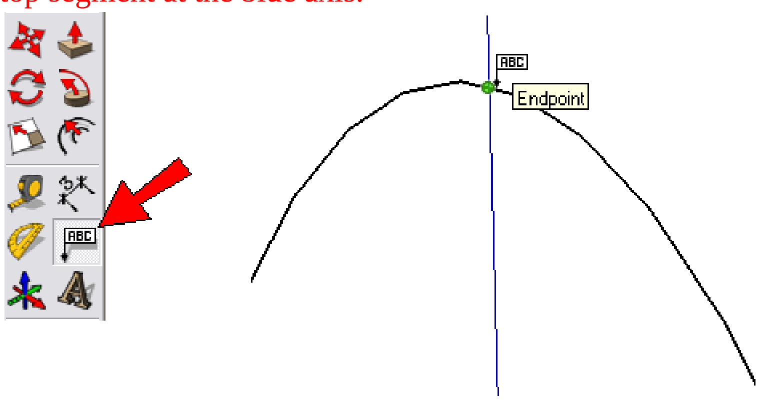
$a = (y - k) / x2$

Before asking SketchUp for coordinates, make sure you'll be getting decimal numbers. Choose **Window / Model Info**, and open the **Units** page. Set the format to **Decimal**. (If you were using **Architectural** units, converting all those feet and inches to "real" numbers would be a real hassle!)

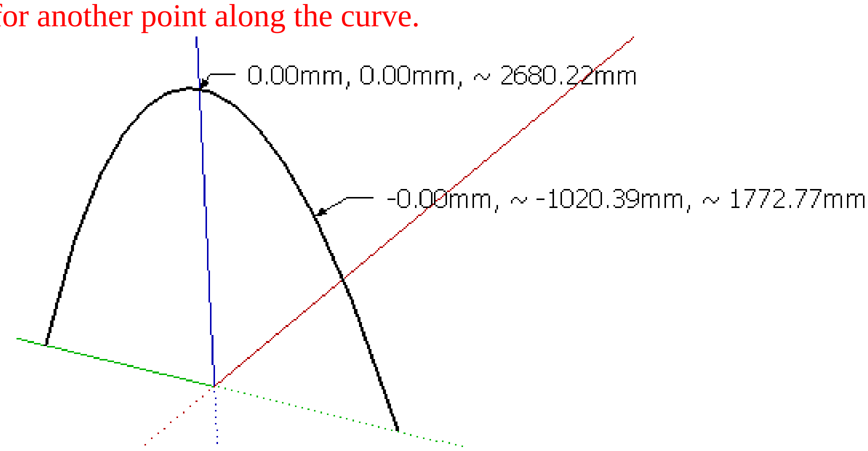


Because of the **Create Group from Slice** option, the parabola was created as a group. You can't get point coordinates on anything that's grouped, so right-click on the parabola and choose **Explode**.

Activate the **Text** tool. To get the vertex coordinates, click the tip of the parabola. Important: You need to click an *endpoint*, as opposed to a midpoint or a point along one of the segments. If there is no endpoint at the very tip, use the Line tool to divide the top segment at the blue axis.



Move the mouse to locate the text, and click to place it. Then click away from the text to finish. Repeat this coordinate labelling for another point along the curve.



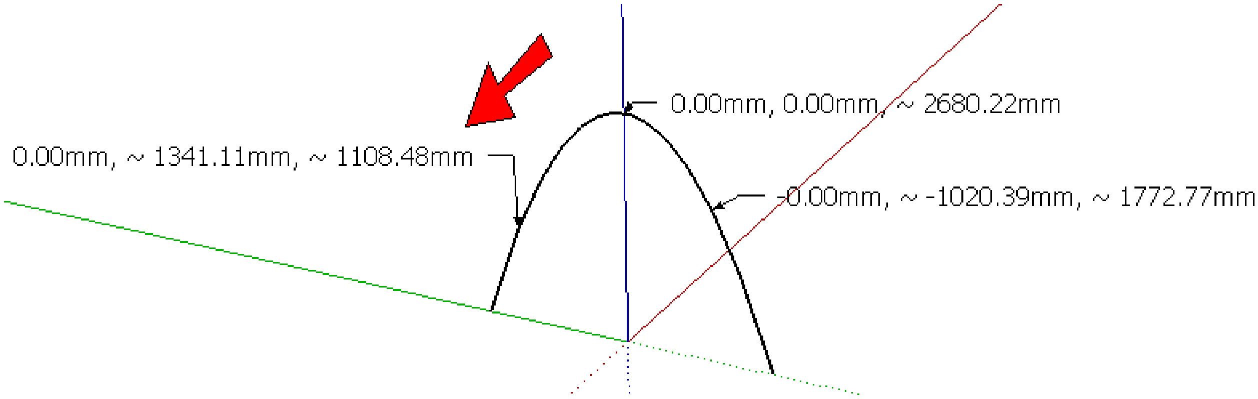
In my example, my vertex coordinates are (h = 0, k = 2680.2).

The coordinates of the other point along the parabola are (1020.4, 1772.8)

Plugging in these numbers into the equation $a = (y - k) / x2$ yields:

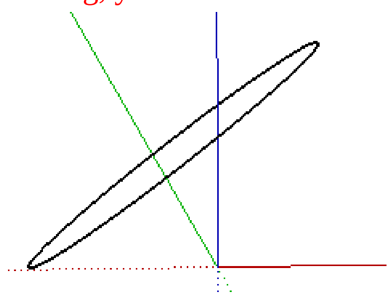
a = -0.0008715

Does this work all along the curve? Let's get the coordinates for another point: (1341.1, 1108.5)



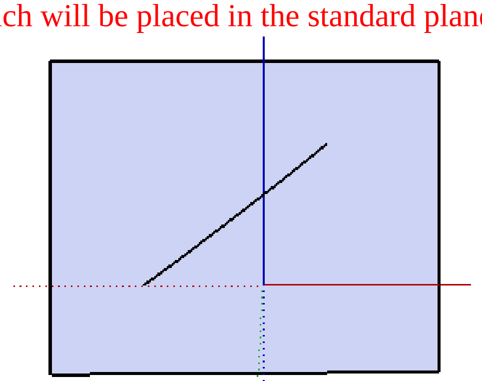
Using the same k value, I get a = -0.0008739. (Not exactly the same as the other a value, but I'd do better if I started with a higher-segmented circle.)

Now for the ellipse. After sectioning and erasing, you should have a tilted ellipse:

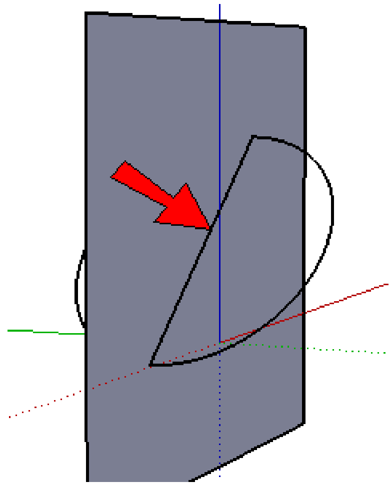


There are a few ways to get this ellipse on a standard plane and centered. Here the way I found to be easiest; you may prefer another method.

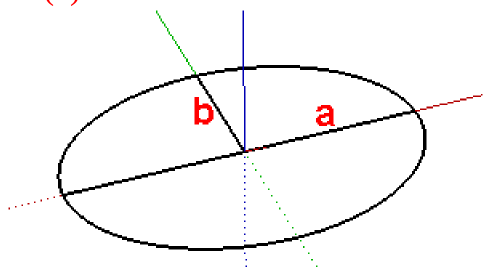
First, face the ellipse head-on, so that you're facing one of the vertical planes (red-blue or red-green). Draw a rectangle surrounding the ellipse, which will be placed in the standard plane you're facing.



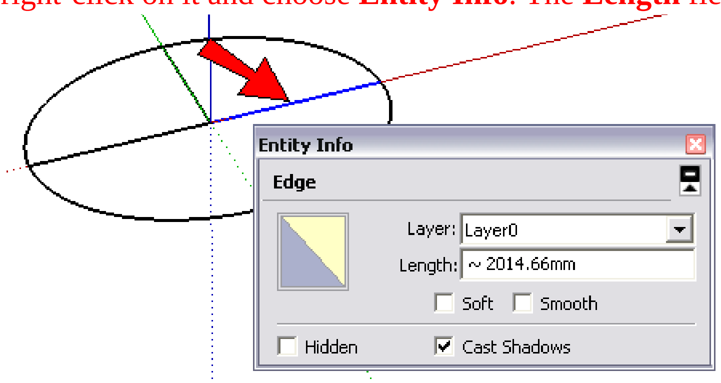
On this rectangular face, draw a line bisecting the ellipse. You'll have to zoom in closely in order to click the exact point where the ellipse intersects the face.



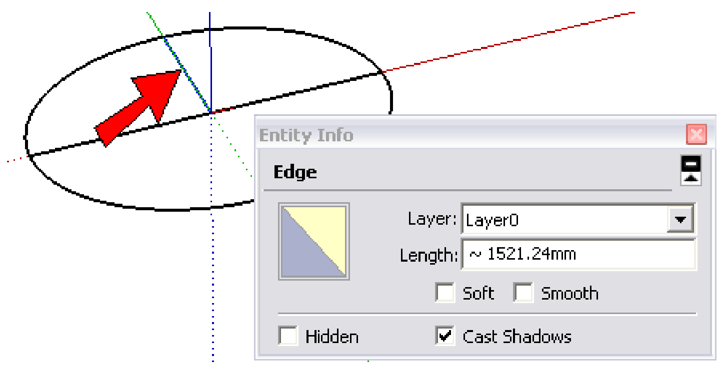
Erase the rectangle, then rotate and center the ellipse in the red-green plane. You already have a line for the major axis (*a*), so add a line for the minor axis (*b*).



To measure the major axis, right-click on it and choose **Entity Info**. The **Length** field tells you, well, the length.



Do the same for the minor axis.

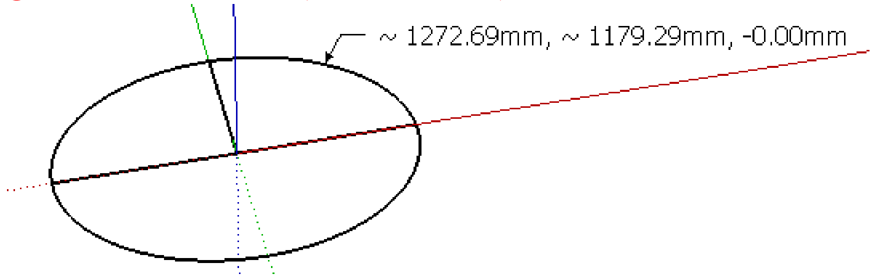


So I have $a = 2014.7$, $b = 1521.2$

The equation of an ellipse is

$(x2 / a2) + (y2 / b2) = 1$

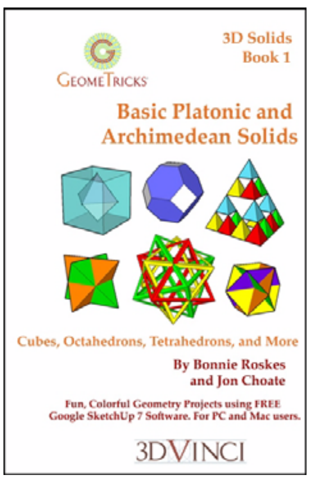
Let's test this by getting a set of coordinates: (1272.7, 1179.3)

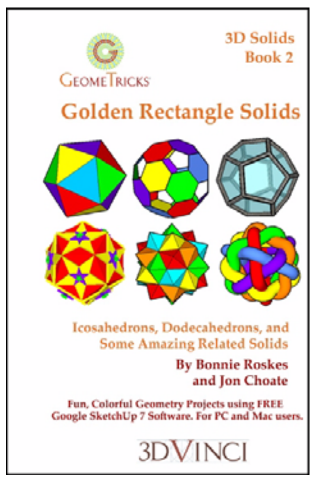


I plug in the numbers into the ellipse equation, and get an answer of almost exactly 1.0!

Want More?

If you like 3D geometry projects in Google SketchUp, you'll love 3DVinci's GeomeTricks series! These are the books in our 3D Solids series:





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